4 $y = e^{x^2+x}$

Show that

$$y'' = y'(2x+1) + 2y$$

ε

Sol'n

$y = e^{x^2+x}$

$y = e^{x^2} \cdot e^{x}$

$y' =$

let $u = e^{x^2}$

let $x^2 = s$

$ds/dx = 2x$

$u = e^{s}$

$du/ds = e^{s}$

$du/dx = du/ds \cdot ds/dx$

$= e^{s} \cdot 2x$

$du/dx = 2xe^{x^2}$

$v = e^{x}$

$dv/dx = e^{x}$

$y' = e^{x} \cdot 2xe^{x^2} + e^{x^2} \cdot e^{x}$

$y' = 2xe^{x^2+x} + e^{x^2+x}$

$y' = e^{x^2+x}(2x+1)$

$u =$

$y' = 2xe^{x^2+x} + e^{x^2+x}$

$v = 2xe^{x^2+x}$

$s = 2x \quad ds/dx = 2$

$J = e^{x^2+x} \quad dJ/dx = 2xe^{x^2+x} + e^{x^2+x}$

$dv/dx = 2x(2xe^{x^2+x} + e^{x^2+x}) + 2e^{x^2+x}$

$dv/dx = 4x^2e^{x^2+x} + 2xe^{x^2+x} + 2e^{x^2+x}$

$y'' = 4x^2e^{x^2+x} + 2xe^{x^2+x} + 2e^{x^2+x} + 2xe^{x^2+x} + e^{x^2+x}$

$= 4x^2e^{x^2+x} + 4xe^{x^2+x} + 3e^{x^2+x}$

$= e^{x^2+x}(4x^2 + 4x + 3)$

$= 4xe^{x^2+x}(x+1$

$y' = e^{x^2+x}(2x+1)$

$u = e^{x^2+x} \quad du/dx = e^{x^2+x}(2x+1)$

$v = (2x+1) \quad dv/dx = 2$

$2e^{x^2+x} + (2x+1)e^{x^2+x}(2x+1)$

$e^{x^2+x}(2x+1) = y'$

$e^{x^2+x} = y$

$y'' = 2y + y'(2x+1)$

$y'' = y'(2x+1) + 2y$

$y^{(n+2)} = y^{(n+1)} \cdot (2x+1) + (n+1)y^{n} \cdot 2$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

$y = x^3 e^{4x}$

$u = x^3$

$v = e^{4x}$

$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} + v^2 + \dots$

$\cancel{y^5 = (x^3)^5}$

$y^5 = (4)^5 e^{4x} \cdot x^3 + 5(4)^4 e^{4x} \cdot 3x^2 + 10(4)^3 e^{4x} \cdot 6x +$

$10(4)^2 e^{4x} \cdot 6 + 0$

$y^5 = 1025 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x} + 0$

$y^5 = e^{4x}(1025x^3 + 3840x^2 + 3840x + 460)$

ii $x^2 d^2y/dx^2 + x\, dy/dx + y = 0$

$x^2 y'' + x y' + y = 0$

using leibnitz theorem

$y^n = y^{(n+2)} x^2 + n \cdot 2x\, y^{n+1} + \frac{2n(n-1)}{2!} y^n + y^{(n+1)} x$

$+ n \cdot y^{(n)\cdot 1} + y^n$

$y^n = x^2 y^{(n+2)} + 2x y^{(n+1)} \cdot n + n(n-1) y^n + x y^{(n+1)} + n y^n + y^n$

$y^n = x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2 - 1) y^{(n)}$

Hence

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2-1) y^{(n)} = 0$