

Petroleum Engineering

1) IF  $y = p^{x^2+x}$

$u = x^2 + x$

$dy/dx = 2x + 1$

$y = p^u$

$dy/du = p^u$

$dy/dx = dy/du \times du/dx$

$= p^u \times 2x + 1$

$2x(1 + 1/p^u) \quad u = x^2 + x$

$dy/dx = 2x + 1 p^{x^2+x}$

$d^2y/dx^2 = 2p^{x^2+x} + 4x^2 + 4x + 1 p^{x^2+x}$

$d^2y/dx^2 = 2p^{x^2+x} + 4x^2 + 4x + 1 p^{x^2+x}$

$y'' = d^2y/dx^2 \quad y' = dy/dx \quad y = p^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2p^{x^2+x} + 4x^2 + 4x + 1 p^{x^2+x}$

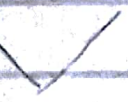
$y'(2x+1) = (2x+1)(2x+1)p^{x^2+x}$

$2y = 2p^{x^2+x}$

$y'(2x+1) + 2y = 2p^{x^2+x} + 4x^2 + 4x + 1 p^{x^2+x}$

$y'' = 2p^{x^2+x} + 4x^2 + 4x + 1 p^{x^2+x}$

$y'' = y'(2x+1) + 2y$



$w_1$

$w_2$

$w_3$

$w_2$

$u = y'' \quad v = 1$

$u' = y'' + 2 \quad v = 0$

$= y'' + 2 - 1 + 0$

$$u^n = y^{n+1} \quad V' = 2$$

$$u^{n+1} = y^n \quad V = 0$$

$$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$$

$$= y^{n+1}(2x+1) + 2n(y^n)$$

$\omega_3$

$$u = y \quad V = 2$$

$$u' = y' \quad V' = 0$$

$$= 2[(y^{0.1}) + 0]$$

$$= 2y^0$$

$$\omega_2 = \omega_2 + \omega_3$$

$$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$$

$$= y^{n+1}(2x+1) + 2(n+1)y^n$$

2a) Using the Leibnitz theorem given that

$$y = x^{3e^{4x}} \text{ determine } y^{(5)}$$

Solution

$$u = e^{4x} \quad V = x^3$$

$$y^{(5)} = u^{(5)}V + 5u^{(4)}V' + 10u^{(3)}V'' + 10u^{(2)}V''' + 5u'V^{(4)} + uV^{(5)}$$

$$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} \cdot 6x) + 5(4^2 e^{4x} \cdot 6)$$

b) + 0

$$= 1024 e^{4x} x^3 + 1280 e^{4x} \cdot 3x^2 + 6720 e^{4x} \cdot 6x + 80 e^{4x} \cdot 6$$

$$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3846 e^{4x} x + 480 e^{4x}$$

1)  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$$x^2 y'' + x y' + y = 0$$

$$\downarrow \quad \quad \downarrow \quad \quad \downarrow$$

$$\omega_1 \quad \quad \omega_2 \quad \quad \omega_3$$

$$\omega_1 + \omega_2 + \omega_3 = 0$$

For  $\omega_1$

$$u = y'' \quad V = x^2$$

$$u' = y'' + 2 \quad V' = 2x$$

$$\begin{aligned}
 u^{n+2} &= y^n & v''' &= 0 \\
 &= y^{(n+2)}(x^2) + n(y^{n+1}) 2x + n(n-1)y^{n-2} x^2 \neq 0 \\
 &= 2x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^{n-2} x^2
 \end{aligned}$$

for  $\omega_2$

$$\begin{aligned}
 u &= y' & v &= x \\
 u^n &= y^{n+1} & v' &= 1 \\
 u^{n-1} &= y^n & v'' &= 0 \\
 &= y^{n+1} \cdot x + n y^n \cdot 0
 \end{aligned}$$

for  $\omega_3$

$$\begin{aligned}
 u &= y & v &= 1 \\
 u^n &= y^n & v' &= 0 \\
 &= y^n \cdot 1
 \end{aligned}$$

$$\omega_1 + \omega_2 + \omega_3 = 0$$

$$\begin{aligned}
 &2x^2 y^{(n+2)} + 2nx y^{n+1} + (n^2 - n) y^n + x y^{n+1} + n y^n + y^n \\
 &x^2 y^{(n+2)} + 2nx y^{n+1} + x y^{n+1} + n^2 y^n - n y^n + x y^n + y^n \\
 &x^2 y^{(n+2)} + 2n+1(x y^{n+1}) + (n^2 + 1) y^n
 \end{aligned}$$