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15/ENG.07/027
Petroleum Engineering

1) IF $y = e^{x^2+x}$

$u = x^2 + x$

$dy/dx = 2x + 1$

$y = e^u$

$dy/du = e^u$

$dy/dx = dy/du \times du/dx$

$= e^u \times 2x + 1$

$2x + 1e^u$ $\qquad u = x^2 + x$

$dy/dx = 2x + 1e^{x^2+x}$

$d^2y/dx^2 = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$d^2y/dx^2 = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = d^2y/dx^2 \quad y' = dy/dx \quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$\underbrace{y''}_{\omega_1} = \underbrace{y'(2x+1)}_{\omega_2} + \underbrace{2y}_{\omega_3}$

ω_1

$u = y'' \qquad v = 1$

$u' = y^{n+2} \qquad v' = 0$

$= y^{n+2} \cdot 1 + 0$

$u^n = y^{n+1}$ $V' = 2$

$u^{n-1} = y^n$ $V = 0$

$= y^{n+1}(2x+1) + n(y^n).2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

ω_3

$u = y$ $V = 2$

$u^n = y^n$ $V' = 0$

$= 2[(y^n.1) + 0]$

$= 2y^n$

$\omega_1 = \omega_2 + \omega_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2a) Using the Leibnitz theorem given that $y = x^3 e^{4x}$ Determine $y^{(5)}$

Solution

$u = e^{4x}$ $V = x^3$

$y^{(5)} = u^{(5)}V + 5u^4V' + 10u^3V'' + 10u^2V^3 + 5u'V^4 + uV^5$

$= (4^5 e^{4x}.x^3 + 5(4^4 e^{4x}.3x^2) + 10(4^3 e^{4x}(6x)) + 5(4^2 e^{4x}$

$6) + 0$

$= 1024e^{4x}x^3 + 1280e^{4x}.3x^2 + 640e^{4x}.6x + 80e^{4x}.6$

$= 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 480e^{4x}$

1) $x^2 dy^2/dx^2 + x\,dy/dx + y = 0$

$x^2 y'' + xy' + y = 0$

$\downarrow$ ω_1 $\downarrow$ ω_2 ω_3

$\omega_1 + \omega_2 + \omega_3 = 0$

For ω_1

$u = y''$ $V = x^2$

$u^n = y^{n+2}$ $V' = 2x$

$u^{n-2} = y^n$ $\quad v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})\,2x + \underline{n(n-1)\,y^n.x} \neq 0$

$= x^2 y^{(n+2)} + 2nx\,(y^{n+1}) + n(n-1)\,y^n \, 2!$

for ω_2

$u = y'$ $\quad u = x$

$u^n = y^{n+1}$ $\quad v = 1$

$u^{n-1} = y^n$ $\quad v'' = 0$

$= y^{n+1}.x + ny^n \neq 0$

for ω_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$= y^n.1$

$\omega_1 + \omega_2 + \omega_3 = 0$

$x^2 y^{n+2} + 2nxy^{n+1} + (n^2 - n)y^n + xy^{n+1} + ny^n + y^n$

$x^2 y^{n+2} + 2nx\,y^{n+1} + xy^{n+1} + n^2y^n - ny^n + ny^n + y^n$

$x^2 y^{n+2} + 2n+1\,(xy^{n+1}) + (n^2+1)\,y^n$