IZOKPU IZOHAN
15/ENG03/018
CIVIL ENGINEERING

~~ENG~~ ENG 281

1. If $y = e^{x^2}$ ---- ①

$v_1 = 1,\ v' = 0$

$u = e^{x^2+x},\ u^n = (2x+1)^n e^{x^2+x}$

$\therefore y' = u^n v + 0$

$y' = (2x+1)e^{x^2+x}$ ---- ②

$y'' = u^n \cdot v + n u^{(n-1)} v' + 0$

$= (2x+1)e^{x^2+x} \cdot (2x+1) + n(2x+1)^{1-1} e^{x^2+x} \cdot 2 + 0$

$y'' = (2x+1)e^{x^2+x} \cdot (2x+1) + (2x+1)^0 e^{x^2+x} \cdot 2$

Substituting eqn ① & ②

$y'' = y'(2x+1) + 2y$

$\therefore y'' = y'(2x+1) + 2y$

Prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Where $W = y''$, from the answer above,

$y'' - y'(2x+1) - 2y = 0$

$W = y''$

$v = 1,\ v' = 0$

$u = y'',\ u^n = y^{(n+2)}$

$W^n = u^n v + 0$

$W^n = y^{(n+2)} + 0$

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$w^{*} = y(2x+1)$

$v = 2x+1, \quad v' = 2, \quad v^2 = 0$

$u = y', \quad u^n = y^{n+1}$

$w^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)\cancel{(n-2)}}{2!} u^{n-2} v^2$

$w^n = y^{(n+1)}(2x+1) + n y^n \cdot 2 + 0$

$= y^{(n+1)}(2x+1) + 2n y^n$

$w = 2y$

$v = 2, \quad v' = 0$

$u = y \quad u^n = y^n$

$w^n = u^n v + 0$

$= y^n \cdot 2$

$= 2y^n$

$\therefore y^n = y^{n+2} - (y^{(n+1)}(2x+1) + 2n y^n) - (2y^n) = 0$

$\therefore y^{n+2} = y^{(n+1)}(2x+1) + 2n y^n + 2y^n$

$\therefore y^{n+2} = (2x+1) y^{(n+1)} + 2y^n (n+1).$

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2 i) $y = x^3 e^{4x}$, find $y^{(5)}$

$u = e^{4x}$, $u^n = 4^n e^{4x}$

$v = x^3$, $v^1 = 3x^2$, $v^2 = 6x$, $v^3 = 6$, $v^4 = 0$

$u^4 = 4$

$$y^n = u^n v + n u^{(n-1)} v^1 + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} \cdot v^4$$

$$y^n = u^n v + n u^{(n-1)} v^1 + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + n(n-1)(n-2) u^{(n-3)} v^3 + 0$$

$$y^5 = 4^5 e^{4x} \times x^3 + 5 \cdot 4^{5-1} e^{4x} \cdot 3x^2 + \frac{5(5-1)}{2!} 4^{5-2} e^{4x} \cdot 6x$$

$$+ \frac{5(5-1)(5-2)}{3!} \cdot 4^{5-3} e^{4x} \cdot 6$$

$$y^5 = 4^5 e^{4x} \cdot x^3 + 5 \times 4^4 e^{4x} \cdot 3x^2 + \frac{5 \times 4 \times 4^3 e^{4x}}{2} \times 6x + \frac{5 \times 4 \times 3 \times 4^2 e^{4x} \times 6}{6}$$

$$y^5 = 1024 e^{4x} \cdot x^3 + 3840 x^2 \cdot e^{4x} + 3840 x e^{4x} + 960 e^{4x}$$

$$y^5 = 64 e^{4x} [16x^3 + 60x^2 + 60x + 15]$$

ii $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

$\therefore x^2y'' + xy' + y = 0$

$W = x^2y''$

$V = x^2, V' = 2x, V'' = 2, V''' = 0$

$u = y, u^n = y^{(n+2)}$

$W^n = u^nv + nu^{n-1}v' + \frac{n(n-1)u^{n-2}v''}{2!} + \frac{n(n-1)(n-2)u^{n-3}v'''}{3!}$

$W^n = y^{(n+2)}\cdot x^2 + ny^{(n+2-1)}\cdot 2x + \frac{n(n-1)y^{(n+2-2)}}{2} \times 2 + 0$

$W^n = x^2y^{n+2} + 2xny^{(n+1)} + n(n-1)y^n$

$W = xy'$

$V = x, v' = 1, v'' = 0$

$u = y', u^n = y^{(n+1)}$

$W^n = u^nv + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}v''}{2!}$

$W^n = y^{(n+1)}x + ny^{(n+1-1)}\cdot 1 + 0$

$W^n = xy^{(n+1)} + ny^{(n)}$

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$W = y$

$V = 1, \; V' = 0$

$u = y, \; u^n = y^n$

$W^n = u^n v + n u^{(n-1)} v'$

$= y^n \cdot 1 + 0$

$W^n = y^n$

$\therefore y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{n+1} + ny^n + y^n$

$y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + ny^n + y^n$

$0 = x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + (n^2 - n + n + 1) \cdot y^n$

$= x^2 y^{(n+1)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$