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15/ENG01/009
CHEMICAL ENGINEERING
ENG 381

1 $y = e^{x^2+x}$

a $y' = (2x+1)\, e^{x^2+x}$

$y'' = (2x+1)(2x+1)\left(e^{x^2+x}\right) + 2\left(e^{x^2+x}\right)$

$y'' = (2x+1)(2x+1)\left(e^{x^2+x}\right) + 2\left(e^{x^2+x}\right)$

and $y \Rightarrow e^{x^2+x}$, $y' = (2x+1)\, e^{x^2+x}$

$\therefore y'' = 2x+1\left(y^{(1)}\right) + 2\left(y^{(0)}\right)$

b $y'' = 2x+1\left(y^{(1)}\right) + 2\left(y^{(0)}\right)$

$y'' - 2x+1\left(y^{(1)}\right) - 2\left(y^{(0)}\right) = 0$

$y^{(2)} - 2x+1\left(y^{(1)}\right) - 2\left(y^{(0)}\right) = 0$

$W_1^{(n)} = y^{(n+2)}$

$W_2^{(n)} = y^{(n+1)}(2x+1) + ny^{n}(2)$

$W_3 = y^{(n)}$

$= y^{(n+2)} - \left[y^{(n+1)}(2x+1) + ny^{n}(2)\right] - 2y^{(n)}$

$0 = y^{(n+2)} - y^{(n+1)}(2x+1) - 2ny^{(n)} - 2y^{(n)}$

$0 = y^{(n+2)} - y^{(n+1)}(2x+1) - \left[2ny^{(n)} + 2y^{(n)}\right]$

$0 = y^{(n+2)} - y^{(n+1)}(2x+1) = 2y^{(n)}[n+1]$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}[n+1]$

$\therefore y^{(n+2)} = 2x+1\left(y^{(n+1)}\right) + 2(n+1)\left[y^{(n)}\right]$

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2a $y = x^3 e^{4x}$

let $u \Rightarrow e^{4x}$

$v = x^3$

From Lebnitz theorem

$$y^{(n)} = u^{(n)}v + nu^{(n-1)}v^1 + \frac{n(n-1)}{2!}u^{(n-2)}v^2 + \cdots$$

$$\cdots + \frac{n(n-1)(n-2)u^{n-3}v^3}{3!} \cdots$$

$u = e^{4x}$ $\quad$ $v = x^3$

$u^{(1)} = 4e^{4x}$ $\quad$ $v^1 = 3x^2$

$u^{(2)} = 16e^{4x}$ $\quad$ $v^2 = 6x$

$u^{(3)} = 64e^{4x}$ $\quad$ $v^3 = 6$

$u^{(4)} = 256e^{4x}$ $\quad$ $v^4 = 0$

$u^{(5)} = 1024e^{4x}$ $\quad$ $v^5 = 0$

$$y^{(5)} = 1024e^{4x}x^3 + 5 \cdot 256e^{4x}\,3x^2 + \cdots$$

$$\cdots + \frac{5(4)64e^{4x}6x}{2!} + \frac{5(4)(3)16e^{4x}6}{3!} + \cdots$$

$$\cdots + \frac{5(4)(3)(2)4e^{4x}(0)}{4!} + \frac{5(4)(3)(2)(1)e^{4x}(0)}{5!}$$

$$y^{(5)} = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x$$

$$+ 960e^{4x}.$$

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b $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$\underset{w_1}{x^2 y^{(2)}} + \underset{w_2}{x y^{(1)}} + \underset{w_3}{y^{(0)}} = 0$

$w_1^{(n)} = x^2 y^{(n+2)} + n y^{(n+1)} 2x + \frac{n(n-1)y^{(n)}}{2!} \cdot 2 + 0 \cdots$

$W_2^{(n)} = x(y^{(n+1)}) + n y^{(n)} + 0$

$W_3^{(n)} = y^{(n)}$

$x^2 y^{(n+2} + n y^{(n+1)} 2x + \frac{n(n-1)y^{(n)}}{2!} \cdot 2 + x(y^{(n+1)}) + n y^{(n)}$

$+ y^{(n)} = 0$

$x^2 y^{(n+2)} + y^{(n+1)} 2xn + n(n-1) y^{(n)} + x(y^{(n+1)}) + n y^{(n)} + y^{(n)}$
$= 0$

$x^2 y^{(n+2)} + y^{(n+1)} [2xn + x] + y^{(n)} [n(n-1) + n + 1] = 0$

$x^2 y^{(n+2)} + y^{(n+1)} [x(2n+1)] + y^{(n)} [n^2 - \cancel{n} + \cancel{n} + 1] = 0$

$x^2 y^{(n+2)} + y^{(n+1)} x [2n+1] + y^{(n)} [n^2+1] = 0$

$\therefore x^2 y^{(n+2)} + x(2n+1) y^{(n+1)} + y^{(n)} [n^2+1] = 0$