[illegible] Daud Olamide

15/ENG07/[illegible]

Petroleum Engineering

If $y = e^{x^2+x}$

$u = x^2 + x$

$\frac{du}{dx} = 2x+1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times 2x+1$

$2x+1e^u \qquad u = x^2+x$

$\frac{dy}{dx} = 2x+1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} \qquad y' = \frac{dy}{dx} \qquad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2+4x+1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1)+2y = 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2+4x+1e^{x^2+x}$

$\underbrace{y''}_{w_1} = \underbrace{y'(2x+1)}_{w_2} + \underbrace{2y}_{w_3}$

w_1

W_2

$u = y'$ $\quad v = 2x+1$

$u^n = y^{n+1}$ $\quad v' = 2$

$u^{n-1} = y^n$ $\quad v'' = 0$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

W_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$= 2[(y^n \cdot 1) + 0]$

$= 2y^n$

$W_1 = W_2 + W_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

Using the leibnitz theorem given that $y = x^3 e^{4x}$ determine $y^{(5)}$

Solution

$u = e^{4x} \quad v = x^3$

$y^{(5)} = u^{(5)}v + 5u^4v' + 10u^3v'' + 10u^2v^3 + 5u'v^4 + uv^5$

$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} \cdot 6x) + 5(4^2 e^{4x} \cdot 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} \cdot 3x^2 + 640 e^{4x} \cdot 6x + 80 e^{4x} \cdot 6$

$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 480 e^{4x}$

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$y^{n-1} = y^{n+1}$ $\quad v'' = 2$

$u^{n-2} = y^{n}$ $\quad v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)\,y^{n}\cdot \cancel{2}}{\cancel{2}!} + 0$

$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)\,y^{n}$

For W_2

$u = y'$ $\quad v = x$

$u^{n} = y^{n+1}$ $\quad v' = 1$

$u^{n-1} = y^{n}$ $\quad v'' = 0$

$= y^{n+1}\cdot x + ny^{n} + 0$

For W_3

$u = y$ $\quad v = 1$

$y^{n} = y^{n}$ $\quad v' = 0$

$= y^{n}\cdot 1$

$W_1 + W_2 + W_3 = 0$

$x^2 y^{n+2} + 2nxy^{n+1} + (n^2 - n)y^{n} + xy^{n+1} + ny^{n} + y^{n}$

$x^2 y^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2 y^{n} - \cancel{ny^{n}} + \cancel{ny^{n}} + y^{n}$

$x^2 y^{n+2} + 2n+1\,(xy^{n+1}) + (n^2+1)\,y^{n}$

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