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 Petroleum Engineering

If $y = e^{u(x)}$
 $u = x^2 + x$
 $\frac{du}{dx} = 2x + 1$

$y = e^u$
 $\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
 $= e^u \times (2x+1)$

$\frac{dy}{dx} = (2x+1)e^{x^2+x}$ $u = x^2 + x$

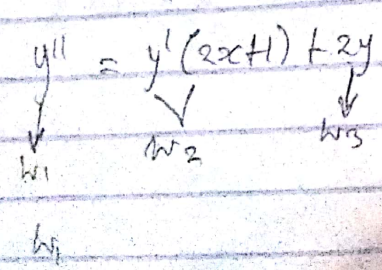
$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2 + 4x + 1 e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2}$ $y' = \frac{dy}{dx}$ $y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$
 $y'' = 2e^{x^2+x} + 4x^2 + 4x + 1 e^{x^2+x}$
 $y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$
 $= 4x^2 + 4x + 1 e^{x^2+x}$

$2y = 2e^{x^2+x}$
 $y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1 e^{x^2+x}$
 $= 2e^{x^2+x} + 4x^2 + 4x + 1 e^{x^2+x}$



w_2

$$u = y^1$$

$$u^n = y^{n+1}$$

$$u^{n+1} = y^n$$

$$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$$
$$= y^{n+1}(2x+1) + 2n(y^n)$$

$$v = 2x+1$$

$$v' = 2$$

$$v = 0$$

w_3

$$u = y$$

$$u^n = y^n$$

$$= 2[(y^n) \cdot 1] + 0$$

$$= 2y^n$$

$$v = 1$$

$$v' = 0$$

$$w_1 = w_2 + w_3$$

$$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$$
$$= y^{n+1}(2x+1) + 2(n+1)y^n$$

Using the Leibnitz theorem given that
 $y = x^3 e^{4x}$ determine $y^{(5)}$

Solution

$$u = e^{4x}$$

$$v = x^3$$

$$y^{(5)} = v^{(5)}u + 5u^4v' + 10u^3v'' + 10u^2v''' + 5u^1v^{(4)} + uv^{(5)}$$

$$= 4^5 e^{4x} x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} \cdot 6x) + 5(4^2 e^{4x} \cdot 6) + e^{4x} \cdot 6$$

$$= 1024 e^{4x} x^3 + 1280 e^{4x} \cdot 3x^2 + 640 e^{4x} \cdot 6x + 80 e^{4x} \cdot 6$$

$$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 480 e^{4x}$$

$$x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} + y = 0$$

$$\begin{aligned}
 u^{n-1} &= (y^{n+1}) & v'' &= 2 \\
 u^{n-2} &= y^n & v''' &= 0 \\
 &= y^{(n+2)} (x^2) + n(y^{n+1}) 2x + n(n-1)y^n \cdot x & & + 0 \\
 & & & \cdot x^1 \\
 &= 5x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n
 \end{aligned}$$

For w_2

$$\begin{aligned}
 u &= y^1 & v &= x \\
 u^n &= y^{n+1} & v' &= 1 \\
 u^{n-1} &= y^n & v'' &= 0 \\
 &= y^{n+1} \cdot x + n y^n \cdot 1
 \end{aligned}$$

For w_3

$$\begin{aligned}
 u &= y & v &= 1 \\
 u^n &= y^n & v' &= 0 \\
 &= y^n \cdot 1
 \end{aligned}$$

$$w_1 + w_2 + w_3 = 0$$

$$\begin{aligned}
 &x^2 y^{n+2} + 2nx y^{n+1} + (n^2 - n) y^n + x y^{n+1} + n y^n + y^n \\
 &x^2 y^{n+2} + 2n x y^{n+1} + x y^{n+1} + n^2 y^n - n y^n + n y^n + y^n \\
 &x^2 y^{n+2} + 2n+1 (x y^{n+1}) + (n^2 + 1) y^n
 \end{aligned}$$