NAME: UNUEBHOLO EMMANUEL OSEREME
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Assignment

Question 1: $y = e^{x^2+x}$

Show that $y'' = y'(2x+1) + 2y$ and prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

$y = e^{x^2+x}$ ——— (1)

$\therefore y^n = C_1 r e^{ax}$ $\quad y' = (2x+1)e^{x^2+x}$ ——— (2)

where $u\frac{du}{dx} + v\frac{dv}{dx}$

$u = 2x+1$; $\frac{du}{dx} = 2$;

$v = e^{x^2+x}$; $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

From equ (1) and (2)

$y'' = y'(2x+1) + 2y$

Let $w_1 = y'$ $\quad w = y'(2x+1)$

$u = y^2$ $\quad$; $v = 1$

$u^n = y(2+n)$

$u = y(1)$ $\quad v = 2x+1$

$u^n = y(1+n)$ $\quad v' = 2$

$u^{n+1} = y^n$

$w_2 = 2y$

$u = y$ $\quad v = 2$

$u^n = y(n)$

$- w(n) = w_2^{(n)} + w_3^{(n)}$

$y^n = C^{(n)}v + n u^{(n-1)}v'$

$y^{(2+n)} = y^{(1+n)} - (2n+1) + n(y(n)).2 + y(n)2$

$y^{(2+n)} = (2n+1)y^{(1+n)} + 2(n+1)y(n)$

Question 2

$y = x^3 e^{4x}$, determine $y^{(5)}$

Solution

$u = e^{4x}$ $\quad v = x^3$

$u^n = 4^n e^{4x}$ $\quad v' = 3x^2$

$u^{n-1} = 4^{n-1} e^{4x}$ $\quad v'' = 6x$

$u^{n-2} = 4^{n-2} e^{4x}$ $\quad v''' = 6$

$u^{n-3} = 4^{n-3} e^{4x}$

$$y^{(n)} = u^n v + n u^{(n-1)} v' + \frac{n(n-1)u^{(n-2)}v''}{2!} + \frac{n(n-1)(n-2)u^{(n-3)}v'''}{3!}$$

$$y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x}) \cdot 3x^2 + \frac{5(4)(4^3 e^{4x})}{2!} \cdot 6x^3 + \cdots +$$

$$\frac{5(4)(3)(4^2 e^{4x}).\cancel{6}}{\cancel{3!}}$$

$y^{(5)} = 1024x^3 + 5840x^2 e^{4x} + 3840x e^{4x} + 96e^{4x}$

$y^{(5)} = 64e^{4x}[16x^3 + 60x^2 + 60x + 15]$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^n = 0$

Solution

Let $w = x^2 y$

$u = y^{(n)}$, $u' = y'''$, $u'' = y^{iv}$ --- $- u^{(n)} = y^{(n+2)}$

$v = x^2$

$v' = 2x$

$v'' = 2$

$v''' = 0$

$w^n = u^n v^0 + \frac{n u^{n-1} v'}{1} + \frac{n(n-1) u^{n-2} v''}{1 \cdot 2} + \frac{n(n-1)(n-2) u^{n-3} v'''}{1 \cdot 2 \cdot 3} + \dots$

$w^n = y^{n+2} x^2 + n \cdot y^{n+1} \cdot 2x + \frac{n(n-1)}{2} y^n \cdot 2 + \cancel{\frac{n(n-1)(n-2)}{6}} y^{n-1} \cdot 0$

$w^n = y^{n+2} \cdot x^2 + n \cdot 2x y^{n+1} + n(n-1) y^n$

Let $w = x y'$

$u = y'$; $u' = y''$; $u^n = y^{(n+1)}$

$v = x$

$v' = 1$

$v'' = 0$

$w^{(n)} = u^n v^0 + \frac{n u^{n-1} v'}{1} + \frac{n(n-1) u^{n-2} v''}{1 \cdot 2}$

$w^{(n)} = y^{n+1} + x + \frac{n y^n}{1} \cdot 1 + \cancel{\frac{n(n-1) y^{n-1}}{1 \cdot 2} \cdot 0}$

$w^{(n)} = y^{n+1} + x + n y^n \cdot 1$

where $w = y$

$w^n = y^n$

$\therefore y^{n+2} x^2 + n \cdot y^{n+1} \cdot 2x + n(n-1) y^n + y^{n+1} + x + n y^n + y^n = 0$

$= y^{n+2} x^2 + [n \cdot 2x y^{n+1} + x y^{n+1}] + [n(n-1) y^n + n y^n + y^n] = 0$

$= x^2 y^{n+2} + (2n+1) x y^{n+1} + [n(n-1) + 1 + n] y^n = 0$

$= x^2 y^{n+2} + (2n+1) x y^{n+1} + (n^2 - n + 1 + n) y^n = 0$

$= x^2 y^{n+2} + (2n+1) x y^{n+1} + (n^2+1) y^n = 0$