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15/ENG07I012

Petroleum-Engineering

ENG 381 Assignment (3)

(1) if $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ and hence, prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

$y = e^{x^2+x}$ - - - - - (1)

$\frac{dy}{dx} = (2x+1)e^{x^2+x} = y'$ - - - (2)

$\frac{d^2y}{dx^2} = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}$

$= (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$ - - (3)

Equating (3) to (2) and (1) eqn

$(4x^2+4x+1)e^{x^2+x} = (4x+4x+1)e^{x^2+x} + 2e^{x^2+x}$ - - (4)

$\therefore y'' = y'(2x+1) + 2y$

$y'' = y'(2x+1) + 2y$

using leibnitz theorem

$-y'' + y'(2x+1) + 2y = 0$

Solving for y''

$u = y''$, $u^n = y^{(n+2)}$

$v^0 = 1$, $v'^0 = 0$

$y^n = y^{n+2}$ to $= y^{n+2}$ ——— (1)

~~$u = y''$~~, ~~$u^n = y^{n}$~~

Solving for $y2x+1$

$u = y''$, $u^n = y^{n+1}$

$v = 2x+1$, $v' = 2$, $v'' = 0$

$y^n = y^{n+1}(2x+1) n y'' 2 + 0$ ——— (2)

Solving for $2y$

$u = y$, $u^n = y^n$

$v = 2$, $v' = 0$

$y^n = 2+0 = 2y^n$ —— (3)

Combining the three eqns

$\therefore\ y^{(n+2)} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{(n)}(n+1)$

(2) Using the leibnitz theorem, given that

i) $y = x^3 e^{4x}$, determine $y^{(5)}$

Soln

$y' = x^3(4e^{4x}) + e^{4x}(3x^2)$

$y' = x^3 4e^{4x} + 3x^2 e^{4x}$ —— (1)

$y'' = x^3(16e^{4x}) + 4e^{4x}3x^2 + 3x^2 4e^{4x} + 6xe^{4x}$

$y''' = x^3 64e^{4x} + 16e^{4x}3x^2 + 3x^2 16e^{4x} + 6x4e^{4x} + 3x^2 16e^{4x} + 6x4e^{4x} + 6x4e^{4x} + 6e^{4x}$

$\cancel{y^{(iv)} = x^3 256e^{4x} + 3x^2 64e^{4x} + 3x^2 64e^{4x} + 6x16e^{4x} + 3x^2 64e^{4x} + 3x^2 64e^{4x} + 24e^{4x} + 24e^{4x} + 0}$

$\cancel{y^{v} = x^3 1024e^{4x} + 3x^2 256e^{4x} + 3x^2 256e^{4x} + 6x64e^{4x} + 3x^2 256e^{4x} + 6x}$

$y^{iv} = x^3 256e^{4x} + 3x^2 64e^{4x} + 3x^2 64e^{4x} + 6x16e^{4x} + 3x^2 64e^{4x} + 3x^2 64e^{4x} + 6x16e^{4x} + 96e^{4x} + 3x^2 64e^{4x} + 6x16e^{4x} + 6x16e^{4x} + 24e^{4x} + 6x16e^{4x} + 24e^{4x} + 24e^{4x} + 0$

$y^{v} = x^3 1024e^{4x} + 3x^2 256e^{4x} + 3x^2 256e^{4x} + 6x64e^{4x} + 3x^2 256e^{4x} + 6x64e^{4x} + 6x64e^{4x} + 96e^{4x} + 3x^2 256e^{4x} + 6x64e^{4x} + 96e^{4x} + 384e^{4x} + 0 + 3x^2 256e^{4x} + 6x64e^{4x} + 6x64e^{4x} + 96e^{4x} + 6x64e^{4x} + 96e^{4x} + 96e^{4x} + 0 + 6x64e^{4x} + 96e^{4x} + 96e^{4x} + 0 + 96e^{4x} + 0$

(ii) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$, show that

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$

Soln

$w = x^2 y''$

Solving for x^2y''

$v = x^2, \; v' = 2x, \; v^2 = 2, \; v^3 = 0$

$u = y'', \; u^n = y(n+2)$

$y^n = y(n+2)x^2 + ny(n+2-1) \cdot 2x + \frac{n(n-1)}{2!} y(n+2-2) \, 2 + 0$

$= y(n+2)x^2 + 2nxy(n-1) + n(n-1)y(n)$

Solving for xy'

$v = x, \; v' = 1, \; v^2 = 0$

$w^n = y(n+1)x + ny(n+1-1) + 0$

$= y(n+1)x + ny^n$

Solving for y

$w = y$

$v = 1, \; v' = 0$

$u = y, \; u^n = y^n$

by eq

$\therefore$ Then $\left[x^2y'' + xy' + y\right]^n = 0$ becomes

$y(n) = x^2 y(n+2) + 2xny(n+1) + xy(n+1) + n(n-1)y^n + ny(n) + y(n)$

$y(n) = x^2 y(n+2) + (2n+1)xy(n+1) + (n^2-n)y(n) + ny(n) + y(n)$

$y(n) = x^2 y(n+2) + (2n+1)xy(n+1) + (n^2-n+n+1)y(n)$

$= x^2 y(n+2) + (2n+1)xy(n+1) + (n^2+1)y(n) = 0$