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15/ENG05/004  
MECHATRONICS

①  $y = e^{2x+n}$   
 $\frac{dy}{dx} = (2n+1)e^{2x+n}$   
 $\frac{d^2y}{dx^2} = 2e^{2x+n} + (2n+1)(2n+1)e^{2x+n}$   
 $= e^{2x+n}((2n+1)(2n+1) + 2)$   
 $y'(2n+1) + 2y$   
 $(2n+1)e^{2x+n}(2n+1) + 2e^{2x+n}$   
 $(2n+1)e^{2x+n}(2n+1) + 2e^{2x+n}$   
 $= e^{2x+n}((2n+1)(2n+1) + 2)$   
 $y'' = y'(2n+1) + 2y$   
 $w_1 \quad w_2 \quad w_3$

$w_1$

$V = y^2$   
 $V^n = y^{(2n)}$

$w_2$

$U = y'$        $V = 2x+1$   
 $U^n = y^{(n+1)}$        $V' = 2$   
 $U^{n-1} = y^{(n)}$        $V'' = 0$

$w_3$

$U = y$        $V = 2$   
 $U^n = y^n$        $V' = 0$

$w_1 = w_2 + w_3$

$y^{n+2} = y^{n+1}(2x+1) + ny^{n-2} + y^n \cdot 2$   
 $= y^{(n+1)}(2x+1) + 2ny^n + 2y^n$   
 $= (2x+1)y^{(n+1)} + 2(n+1)y^n$

②  $y = x^3 e^{4x}$   
 $U = e^{4x}$        $V = x^3$   
 $U^n = 4^n e^{4nx}$        $V' = 3x^2$   
 $U^{n-1} = 4^{(n-1)} e^{4(n-1)x}$        $V'' = 6x$   
 $U^{n-2} = 4^{(n-2)} e^{4(n-2)x}$        $V''' = 6$   
 $U^{n-3} = 4^{(n-3)} e^{4(n-3)x}$        $V^{(4)} = 0$   
 $= 4^n e^{4nx} x^3 + n 4^{(n-1)} e^{4(n-1)x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{(n-2)} e^{4(n-2)x}$   
 $6x + \frac{n(n-1)(n-2)}{2!} 4^{(n-3)} e^{4(n-3)x} \cdot 6$   
 $n = 5$   
 $= 4^5 e^{4nx} x^3 + 5 \cdot 4^4 e^{4nx} \cdot 3x^2 + 10 \cdot 4^3 e^{4nx} \cdot 6x + 6 \cdot 4^2 e^{4nx} \cdot 6$   
 $e^{4x} \cdot 6$

⑤  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

Show that  $x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^{(n)} = 0$   
 $x^2 y^{(2)} + x y^{(1)} + y = 0$   
 $w_1 \quad w_2 \quad w_3$

$w_1$

$U = y^{(2)}$        $V = x^2$   
 $U^n = y^{(n+2)}$        $V' = 2x$   
 $U^{(n-1)} = y^{(n+1)}$        $V'' = 2$   
 $U^{(n-2)} = y^{(n)}$        $V''' = 0$

$w_2$

$U = y^{(1)}$        $V = x$   
 $U^n = y^{(n+1)}$        $V' = 1$   
 $U^{n-1} = y^{(n)}$        $V'' = 0$

$w_3$

$$u = y$$

$$u'' = y''$$

$$w_1 + w_2 + w_3$$

$$= y^{n+2} \cdot x^2 + ny^{n+1} \cdot 2x + \frac{n(n-1)}{2!} y^n \cdot 2 + y^{n+1} \cdot x + ny^n + y^n = 0$$

$$= x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$$

$$= x^2 y^{(n+1)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + ny^n + y^n$$

$$= x^2 y^{(n+1)} + xy^{(n+1)} (2n+1) + y^n (n(n-1) + n+1)$$

$$= x^2 y^{(n+1)} + xy^{(n+1)} (2n+1) + y^n (n^2 - n + n + 1)$$

$$= x^2 y^{(n+1)} + xy^{(n+1)} (2n+1) + y^n (n^2 + 1) = 0$$