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15/ ENG06/011

ENG 381.

1. (1) $y = e^{x^2+x}$

show that

$y'' = y'(2x+1) + 2y.$

soln.

$y = e^{x^2+x}$

$\frac{dy}{dx} = y'$

let $u = x^2 + x$

$\frac{du}{dx} = 2x+1$

$\therefore y = e^u$; $dy/dx = y' = dy/du \times du/dx$

$\frac{dy}{dx} = y' = e^u \times 2x+1$

$y' = (2x+1)e^u$

$y' = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = y''$

$y' = (2x+1)e^{x^2+x}$

using product rule,

let $u = 2x+1$ $\qquad v = e^{x^2+x}$

$\frac{du}{du} = 2.$ $\qquad$ using chain rule,

$\frac{dv}{dx} = 2x+1\, e^{x^2+x}$.

$\therefore \frac{d^2y}{dx^2} = y'' = v\, du/dx + u\, dv/dx.$

$= 2(e^{x^2+x}) + 2x+1(2x+1e^{x^2+x})$

Since, $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$\therefore y'' = 2(y) + (2x+1)(y').$

$y'' = 2y + (2x+1)y'$

$y'' = y'(2x+1) + 2y$

b) Hence prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$$

Soln.

$y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

using leibnitz theorem;

i. y''

$u = y''$; $v = 1$

$u^n = y^{n+2}$ $v' = 0$

$= u^n v + n u^{n-1} v'$

$= y^{n+2}(1) + n(y^{n+1})(0)$

$= y^{n+2}$

ii. $y'(2x+1)$

$u = y'$ $v = -(2x+1)$

$u^n = y^{n+1}$ $v' = -2$

$v'' = 0$

$= u^n v + n u^{n-1} v' + n(n-1)u^{n-2} v''$

$= -y^{n+1}(2x+1) + n(y^n)(-2)$

$= -(2x+1)y^{n+1} - 2ny^n$

iii $-2y$

$u = y$ $v = -2$

$u^n = y^n$ $v' = 0$

$= u^n v + n u^{n-1} v'$

$= -2y^n$

$\therefore y^{n+2} - (2x+1)y^{n+1} - 2ny'' - 2y^n = 0$

$\therefore y^{n+2} = (2x+1)y^{n+1} + 2ny'' + 2y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + 2y^n(n+1)$

$\therefore y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

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2.i. $y = x^3 e^{4x}$.

y^5.

$v = x^3 \quad u = e^{4x}$

$v' = 3x^2 \quad u' = 4e^{4x}$

$v^2 = 6x \quad u^2 = 16e^{4x}$

$v^3 = 6 \quad u^3 = 64e^{4x}$

$v^4 = 0 \quad u^4 = 256e^{4x}$

$v^5 = 0 \quad u^5 = 1024e^{4x}$

$$y^5 = u^5 v + n u^4 v' + \frac{n(n-1)}{2!} u^3 v^2 + \frac{n(n-1)(n-2)}{3!} u^2 v^3 + \frac{n(n-1)(n-2)(n-3)}{4!} u' v^4$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u v^5$$

$$y^5 = [1024e^{4x}(x^3)] + [5(256e^{4x})3x^2] + [\frac{5\times4}{2} \times 64e^{4x} \times 6x]$$
$$+ [\frac{5\times4\times3}{3\times2} \times 16e^{4x} \times 6] + [0][0].$$

$$y^5 = 1024e^{4x}x^3 + 1280e^{4x}(3x^2) + 640e^{4x}(6x) + 160e^{4x}(6).$$

$$y^5 = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x}$$

$$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960).$$

ii). $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

$x^2 y'' + xy + y = 0.$

a. $x^2 y''$

$u = y'' \quad v = x^2$

$u^n = y^{n+2} \quad v' = 2x$

$v'' = 2$

$v''' = 0.$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'$$

$$y^n = y^{n+2}x^2 + 2xny^{n+1} + n(n-1)y^n.$$

3) xy'

$u = y' \quad v = x$

$u^n = y^{n+1} \quad v' = 1$

$v'' = 0.$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v''$$

$y^n = y^{n-1} x + n y^n (1) + 6$

$y^n = x y^{n+1} + n y^n$

(b) y

$u = y \quad v = 1$

$u^n = y^n \quad v' = 0$

$y^n = u^n v + n u^{n-1} v'$

$= y^n (1) + n(y^{n-1})(0)$

$= y^n$

$y^n = y^{n+2} x^2 + n y^{n+1} 2x + [n(n-1) y^n] + x y^{n+1} + n y^n + y^n.$

$y^n = (n^2 - n + n + 1) y^n + (2xn + x) y^{n+1} + x^2 (y^{n+2}).$

$0 = x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n$

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0.$

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