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COURSE: ENG381

1 If $y = e^{x^2+x}$ show that $y = y(2x+1) + 2y$ and hence, prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$W_1 = y''$

$u' = y''$ $u^n = y^{n+2}$

$W_2 = y'(2x+1)$ $v = 2x+1$

$u = y'$ $v' = 2$

$u^n = y^{n+1}$ $v'' = 0$

$y^n = u^n v + nu^{n-1}v'$

$= y^{n+1}(2x+1) + ny^{n-1+1}$

$= y^{n+1}(2x+1) + ny^n \cdot 2$

$W_3 = 2y$

$v = 2$ $v' = 0$

$u^n = y^n$

$y^n = u^n v$

$= 2y^n$

$W_1 = W_2 + W_3$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^{(n)} + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

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2. $y = x^3 e^{4x}$ determine $y^{(5)}$

$u = e^{4x}$ $V = x^3$

$u^n = 4^n e^{4x}$ $V' = 3x^2$

$u^{(n-1)} = 4^{n-1} e^{4x}$ $V^2 = 6x$

$u^{(n-2)} = 4^{n-2} e^{4x}$ $V^3 = 6$

$u^{(n-3)} = 4^{n-3} e^{4x}$ $V^4 = 0$

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} \cdot v^4$$

$$y^n = 4^n e^{4x} x^3 + n 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x +$$

$$\frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} \cdot 6$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x} 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$y^5 = 4^5 e^{4x} x^3 + 5 \times 4^4 e^{4x} 3x^2 + 5(5-1) 4^3 e^{4x} 3x + 5(5-1)(5-2) 4^2 e^{4x}$$

$$y^5 = 1024 e^{4x} x^3 + 3840 x^2 e^{4x} + 3840 + 960 e^{4x}$$

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

$x^2y'' + xy + y = 0$

$w = x^2y''$ $\quad v = x^2 \quad v' = 2x \quad v'' = 2 \quad v''' = 0$

$u = y^n \quad u^n = y^{n+2}$

$$y^n = u^n v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \frac{n(n-1)(n-2)}{3!}u^{(n-3)}v'''$$

$$= y^{n+2}\cdot x^2 + ny^{(n+2-1)}\cdot 2x + \frac{n(n-1)}{2!}y^{(n-2+2)}\cdot 2 + \frac{n(n-1)(n-2)}{3!}y^{(n-3+2)}\cdot 0$$

$$= x^2y^{n+2} + ny^{n+1}\cdot 2x + \frac{n(n-1)}{2}y^n\cdot 2 + \frac{n(n-1)(n-2)}{3}y^{(n-1)}\cdot 0$$

$$= x^2y^{n+2} + 2xny^{n+1} + 2\frac{n(n-1)}{2}y^n$$

$w = xy$

$v = x \quad v' = 1 \quad v'' = 0$

$u = y \quad u^n = y^{n+1}$

$$y^n = u^n v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v''$$

$$y^n = y^{n+1}\cdot x + ny^{(n-1+1)}\cdot 1 + \frac{n(n-1)}{2!}y^{(n-2+1)}\cdot 0$$

$$y^n = xy^{n+1} + ny^n$$

$w = y$

$u = y \quad u^n = y^n \quad v = 1 \quad v'' = 0$

$y^n = u^n v$

$y^n = y^n\cdot 1 + 0 = y^n$

$$y^n = x^2y^{n+2} + 2xny^{n+1} + \cancel{2}\frac{n(n-1)}{\cancel{2}}y^n + xy^{n+1} + ny^n + y^n$$

$$= x^2y^{n+2} + (2xn + x)y^{n+1} + (n(n-1) + n + 1)y^n = 0$$

$$= x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2 - n + n + 1)y^n = 0$$

$$x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$$