

1)  $y = e^{x^2+x}$

$\frac{dy}{dx} = (2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= e^{x^2+x} [(2x+1)(2x+1) + 2]$

$y' (2x+1) + 2y$

$(2x+1)e^{x^2+x} (2x+1) + 2y$

$(2x+1)e^{x^2+x} (2x+1) + 2e^{x^2+x}$

$= e^{x^2+x} [(2x+1)(2x+1) + 2]$

$y'' = y' (2x+1) + 2y$

$w_1 \quad w_2 \quad w_3$

w1

$v = y^2$

$u^n = y^{(n+2)}$

w2

$u = 0' \quad v = 2x+1$

$u^n = y^{n+1} \quad v' = 2$

$u^{n-1} = y^{n+1} \rightarrow 0 \quad v'' = 0 \rightarrow 0$

w3

$u = y \quad v = 2$

$u^n = y^n \quad v = 0$

$w_1 = w_2 + w_3$

$y^{n+2} = y^{n+1}(2x+1) + ny^n \cdot 2 + y^{n+2}$

$= y^{n+1}(2x+1) + 2ny^n + 2y^{n+2}$

$= (2x+1)y^{n+1} + 2(n+1)y^n$

2.  $y = x^3 e^{4x}$

$u = e^{4x}$

$v = x^3$

$u^n = 4^n e^{4nx}$

$v^{(1)} = 3x^2$

$u^{n-1} = 4^{(n-1)} e^{4(x-1)}$

$v^{(2)} = 6x$

$u^{n-2} = 4^{(n-2)} e^{4x}$

$v^{(3)} = 6$

$u^{n-3} = 4^{n-3} e^{4x}$

$= 4^n e^{4x} x^3 + n 4^{(n-1)} e^{4x} \cdot 3x^2 + n(n-1)$

2!

$4^{(n-1)} e^{4x} \cdot 6x + n(n-1)(n-2) 4^{n-3}$

3!

$e^{4x} \cdot 6$

$n=5$

$= 4^5 e^{4x} x^3 + 5 \cdot 4^4 e^{4x} \cdot 3x^2 + 10 \cdot 4^3 e^{4x}$

$6x + 10 \cdot 4^2 e^{4x} \cdot 6$

3)  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

show that  $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)}$

$+ (n^2+1) y^{(n)} = 0$

$$2x^2 y^{(2)} + 2xy^{(1)} + y = 0$$

$w_1 \quad w_2 \quad w_3$

w<sub>1</sub>

$$u = y^{(2)} \quad v = x^2$$

$$u^n = y^{n+2} \quad v' = 2x$$

$$u^{(n-1)} = y^{(n+1)} \quad v'' = 2$$

$$u^{(n-2)} = y^{(n)}$$

w<sub>2</sub>

$$u = y^{(1)} \quad v = x$$

$$u^n = y^{n+1} \quad v' = 1$$

$$u^{n-1} = y^n \quad v'' = 0$$

w<sub>3</sub>

$$u = y$$

$$u^n = y^n$$

$$w_1 + w_2 + w_3$$

$$= y^{n+2} \cdot x^2 + n y^{n+1} \cdot 2x +$$

$$\frac{n(n-1)}{2!} y^n \cdot 2 + y^{n+1} \cdot x + n y^n + y^n$$

$$= 0$$

$$x^2 y^{(n+2)} + 2x n y^{(n+1)} + n(n-1) y^n +$$

$$x y^{n+1} + n y^n + y^n = 0$$

$$x^2 y^{(n+2)} + 2x n y^{(n+1)} + n y^{(n+1)} +$$

$$n(n-1) y^n + n y^n + y^n$$

$$= x^2 y^{(n+2)} + 2x y^{(n+1)} (2n+1) + y^n$$

$$n(n-1) + n + 1$$

$$= x^2 y^{(n+2)} + 2x y^{(n+1)} (2n+1) + y^n$$

$$n(n-1) + n + 1$$

$$= x^2 y^{(n+2)} + 2x y^{(n+1)} (2n+1) + y^n (n^2 +$$

$$= 0$$