ADEDIPE E SEGUN
15/ENG06/001
MECHANICAL ENGINEERING

1. If $y = e^{x^2+x}$
Show that
$y'' = y'(2x+1) + 2y$
$y = e^{x^2+x}$
$\frac{dy}{dx} = y'$
Let $y = e^u$, $dy/dx = y' = dy/du \times du/dx$
$\frac{dy}{dx} = y' = e^u \times 2x+1$

$y' = (2x+1)e^u$
$y' = (2x+1)e^{x^2+x}$
$\frac{d^2y}{dx^2} = y''$
$y' = (2x+1)e^{x^2+x}$
using product rule.
Let $u = 2x+1$ $\qquad v = e^{x^2+x}$
$\frac{du}{dx} = 2$ $\qquad \frac{dv}{dx} = 2x+1\, e^{x^2+x}$
$\therefore \frac{d^2y}{dx^2} = y'' = v\, du/dx + u\, dv/dx$
$= 2(e^{x^2+x}) + 2x+1(2x+1\, e^{x^2+x})$
Since $y = e^{x^2+x}$
$y' = (2x+1)e^{x^2+x}$
$y'' = 2(y) + (2x+1)(y')$
$y'' = 2y + (2x+1)y'$
$y'' = y'(2x+1) + 2y$

b. Hence prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$
$y'' = y'(2x+1) + 2y$
$y'' - y'(2x+1) - 2y = 0$
using Leibnitz theorem.

y''

$u = y''$ $\quad v = 1$

$u^n = y^{n+2}$ $\quad v' = 0$

$= u^n v + n u^{n-1} v'$

$= y^{n+2}(1) + n(y^{n+1})(0)$

$= y^{n+2}$

$-y'(2x+1)$

$u = y'$ $\quad v = -(2x+1)$

$u^n = y^{n+1}$ $\quad v' = -2$

$v'' = 0$

$= u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v''$

$= -y^{n+1}(+2x+1) + n(y^n)(-2)$

$= -(2x+1)y^{n+1} - 2ny^n$

3. $-2y$

$u = y$ $\quad v = -2$

$u^n = y^n$ $\quad v' = 0$

$= u^n v + n u^{n-1} v'$

$= -2y^n$

$\therefore y^{n+2} - (2x+1)y^{n+1} - 2ny^n - 2y^n = 0$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + 2y^n(n+1)$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

(2) using the Leibnitz theorm, given that

1. $y = x^3 e^{4x}$

determine $y^{(5)}$

$v = x^3$ $\quad u = e^{4x}$

$v' = 3x^2$ $\quad u' = 4e^{4x}$

$v^{(2)} = 6x$ $\quad u^{(2)} = 16e^{4x}$

$v^{(3)} = 6$ $\quad u^{(3)} = 64e^{4x}$

$v^{(4)} = 0$ $\quad u^{(4)} = 256e^{4x}$

$v^{(5)} = 0$ $\quad u^{(5)} = 1024e^{4x}$

$y^5 = u^5v + nu^4v^1 + \frac{n(n-1)}{2!}u^3v^2 + \frac{n(n-1)(n-2)}{3!}u^2v^3 + \frac{n(n-1)(n-2)(n-3)}{4!}$

$u^1v^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}uv^5$

$y^5 = [1024e^{4x}(x^3)] + [5(256e^{4x})3x^2] + [\frac{5\times4}{2}\times 64e^{4x}\times 6x]$

$+ [\frac{5\times4\times3}{3\times2}\times 16e^{4x}\times 6] + [0][0]$.

$y^5 = 1024e^{4x}x^3 + 1280e^{4x}(3x^2) + 640e^{4x}(6x) + 160e^{4x}(6)$

$y^5 = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x}$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

1) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy + y = 0$

2) x^2y''

$u = y''$ $\qquad v = x^2$

$u^n = y^{n+2}$ $\qquad v' = 2x$

$v'' = 2$

$v''' = 0$

$y^n = u^nv + nu^{n-1}v' + n(n-1)u^{n-2}v'' + \frac{n(n-1)(n-2)}{3!}u^{n-3}v'''$

$y^n = y^{n+2}x^2 + ny^{n+1}(2x) + \frac{n(n-1)}{2!}y^n(2) + 0$

$y^n = y^{n+2}x^2 + 2xny^{n+1} + n(n-1)y^n$

3) xy'

$u = y'$ $\qquad v = x$

$u^n = y^{n+1}$ $\qquad v' = 1$

$v'' = 0$

$y^n = u^nv + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v''$

$y^n = y^{n+1}x + ny^n(1) + 0$

$y^n = xy^{n+1} + ny^n$

3.

y

$u = y$

$u^n = y^n$ $\qquad\qquad v = 1$

$\qquad\qquad\qquad\qquad v' = 0$

$y^n = u^n v + n u^{n-1} v'$

$= y^n(1) + n(y^{n-1})(0)$

$= y^n$

$y^n = y^{n+2} x^2 + n y^{n+1} 2x + (n(n-1)) y^n + x y^{n+1} + n y^n + y^n$

$y^n = x^2(y^{n+2}) + 2xn(y^{n+1}) + (n^2 - n) y^n + x(y^{n+1}) + n y^n + y^n$

$y^n = (n^2 - n + n + 1) y^n + (2xn + x) y^{n+1} + x^2(y^{n+2})$

$0 = x^2 y^{(n+2)} + (2xn + x) y^{n+1} + (n^2 + 1) y^n$

$0 = x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n$

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$