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15/ENG05/009
Civil Engr

(1) If $y = e^{x^2+x}$

$U = x^2 + x$

$dy/dx = 2x + 1$

$y = e^u$

$dy/du = e^u$

$dy/dx = dy/du \times du/dx$

$= e^u \times 2x+1$

$2x+1e^u \quad U = x^2+x$

$dy/dx = 2x+1e^{x^2+x}$

$d^2y/dx = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} \quad y' = \frac{dy}{dx} \quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + 1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$\underset{W_1}{y''} = \underset{W_2}{y'(2x+1)} + \underset{W_3}{2y}$

W_1

$U = y'' \quad , V = 1$

$U^n = y^{n+2} \quad V = 0$

$= y^{n+2} \cdot 1 + 0$

W_2

$U = y' \quad V = 2x+1$

$u^n = y^{n+1}$ $\quad v' = 2$

$u^{n-1} = y^n$ $\quad v = 0$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

W_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$= 2[(y^n \cdot 1) + 0]$

$= 2y^n$

$W_1 = W_2 + W_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

(2a) Using the leibnitz theorem given that $y = x^3e^{4x}$, determine $y^{(5)}$

<u>Soln</u>

$u = e^{4x}$ $\quad v = x^3$

$y^{(5)} = u^{(5)}v' + 5u^4v' + 10u^3v'' + 10u^2v^5 + 5u'v^4 + uv^3$

$= 4^5 e^{4x} \cdot x^3 + 5(4^4e^{4x} \cdot 3x^2) + 10(4^3e^{4x}\, 6x) + 5(4^2e^{4x} \cdot 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} \cdot 3x^2 + 640e^{4x} \cdot 6x + 80e^{4x} \cdot 6$

$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 480 e^{4x}$.

(2b) $\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$\downarrow \quad \downarrow \quad \downarrow$

$W_1 \quad W_2 \quad W_3$

$W_1 + W_2 + W_3 = 0$

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$u^{n-1} = y^{n+1}$ $\qquad v'' = 2$

$u^{n-2} = y^{n}$ $\qquad v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})\,2x + \dfrac{n(n-1)y^{n}\cdot 2}{2!} + 0$

$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^{n}$

For w_2

$u = y'$ $\qquad v = x$

$u^{n} = y^{n+1}$ $\qquad v' = 1$

$u^{n-1} = y^{n}$ $\qquad v'' = 0$

$= y^{n+1}\cdot x + ny^{n} + 0$

For w_3

$u = y$ $\qquad v = 1$

$y^{n} = y^{n}$ $\qquad v' = 0$

$= y^{n}\cdot 1$