

(i) If $y = e^{x^2+x}$

$u = x^2+x$

$dy/dx = 2x+1$

$y = e^u$

$dy/du = e^u$

$dy/dx = dy/du \times du/dx$

$= e^u \times 2x+1$

$2x+1 e^u \quad u = x^2+x$

$dy/dx = 2x+1 e^{x^2+x}$

$d^2y/dx^2 = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} \quad y' = \frac{dy}{dx} \quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

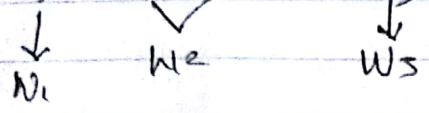
$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$
 $= 4x^2 + 4x + 1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$
 $= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = y'(2x+1) + 2y$



W1

$u = y'' \quad , \quad v = 1$

$u^n = y^{n+2}$

$v = 0$

$= y^{n+2} \cdot 1 + 0$

W2

$u = y'$

$v = 2x+1$

$$u^n = y^{n+1}$$

$$u^{n-1} = y^n$$

$$= y^{n+1} (2x+1) + n(y^n) = 2 + 0$$

$$= y^{n+1} (2x+1) + 2n(y^n)$$

$$v' = 2$$

$$v = 0$$

W_2

$$u = y$$

$$v = 1$$

$$u^n = y^n$$

$$v' = 0$$

$$= 2 [(y^n \cdot 1) + 0]$$

$$= 2y^n$$

$$W_1 = W_2 + W_3$$

$$y^{n+2} = y^{n+1} (2x+1) + 2n(y^n) + 2y^n$$

$$= y^{n+1} (2x+1) + 2(n+1)y^n$$

(2a) Using the Leibnitz theorem given that $y = x^3 e^{4x}$, determine $y^{(5)}$

Soln

$$u = e^{4x} \quad v = x^3$$

$$y^{(5)} = u^{(5)}v + 5u^{(4)}v' + 10u^{(3)}v'' + 10u^2v^{(3)} + 5u'v^{(4)} + uv^{(5)}$$

$$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} \cdot 6x) + 5(4^2 e^{4x} \cdot 6) + 0$$

$$= 1024 e^{4x} x^3 + 1280 e^{4x} \cdot 3x^2 + 640 e^{4x} \cdot 6x + 80 e^{4x} \cdot 6$$

$$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 480 e^{4x}$$

(3)

$$\frac{x^2 dy}{dx^2} + \frac{xy}{dx} + y = 0$$

$$x^2 y'' + xy' + y = 0$$

$$\downarrow$$

W_1

$$\downarrow$$

W_2

$$\downarrow$$

W_3

$$W_1 + W_2 + W_3 = 0$$

$$u^{n+1} = y^{n+1}$$

$$v'' = x$$

$$u^{n+2} = y^n$$

$$v''' = 0$$

$$= y^{n+2} (x^2) + n(y^{n+1}) \frac{2x + n(n-1)y^n \cdot x}{2!} + 0$$

$$= x^2 y^{n+2} + 2nx(y^{n+1}) + n(n-1)y^n$$

For w_2

$$u = y'$$

$$v = x$$

$$u^n = y^{n+1}$$

$$v' = 1$$

$$u^{n-1} = y^n$$

$$v'' = 0$$

$$= y^{n+1} \cdot x + ny^n + 0$$

For w_3

$$u = y$$

$$v = 1$$

$$y^n = y^n$$

$$v' = 0$$

$$= y^n \cdot 1$$