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Assignment 3.

Course :- ENG 381 K Engineering Mathematics II ?

1) If $y = e^{x^2+x}$, Show that $y = y(2x+1) + 2y$ and hence Prove that $y^{(n+2)} = (2x+1)\, y^{(n+1)} + 2(n+1)y^n$

Soln.

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

$y' = (2x+1)e^{x^2+x}$

$y'(2x+1) = (2x+1)e^{x^2+x}(2x+1)$

$= (2x+1)^2 e^{x^2+x}$

$2y = 2 \cdot e^{x^2+x}$

$\therefore y'(2x+1) + 2y = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$2e^{x^2+2} + (2x+1)^2 e^{x^2+x} = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$\therefore w = y''$

$w^n = y^n$

$P = y'(2x+1)$

$\therefore v = 2x+1$ $\quad u = y'$

$v' = 2$ $\quad u^n = y^{n+1}$

$v'' = 0$

$p^n = y^{n+1} \cdot (2x+1) + n \cdot y^n \cdot 2$

$S = 2y$

$S^n = 2y^n$

$w^n = p^n + s^n$

$y^{n+2} = (2x+1)\, y^{n+1} + 2n y^n + 2y^n$

$y^{n+2} = (2x+1)\, y^{n+1} + 2(n+1)\, y^n$

2) Using the leibnitz theorem, given that

i) $y = x^3 e^{4x}$. determine $y^{(5)}$

ii) $x^2 \cdot \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Soln.

i) $y = x^3 e^{4x}$

$V = x^3$ $\quad U = e^{4x}$

$V' = 3x^2$ $\quad U' = 4e^{4x}$

$V'' = 6x$ $\quad U'' = 4 \cdot 4e^{4x}$ $\quad r = 0$ to $r = 3$

$V''' = 6$ $\quad U''' = 4 \cdot 4 \cdot 4 e^{4x}$

$V^{iv} = 0$ $\quad U^n = U^n e^{4x}$

$$\therefore y^n = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x$$
$$+ \frac{n(n-1)(n-2)}{6} 4^{n-3} e^{4x} \cdot 6$$

$$y^n = 4^n e^{4x} x^3 + n4^{n-1} e^{4x} 3x^2 + n(n-1)4^{n-2} e^{4x}\, 3x$$
$$+ n(n-1)(n-2)\, 4^{n-3} e^{4x}$$

$$y^5 = 4^5 e^{4x} x^3 + 4e^{4x}\, 3x^2 + 5 \cdot 4 \cdot 4^3 e^{4x}\, 3x + \frac{5 \cdot 4 \cdot 3 \cdot 4^2 e^{4x}}{2}$$

ii) $x^2\, d^2y/dx^2 + x\, dy/dx + y = 0$

It can be re-written as

$$x^2 y'' + xy' + y = 0$$

Let $w = x^2 y''$

$V = x^2$ $\quad U = y''$

$V' = 2x$ $\quad U' = y'''$

$V'' = 2$ $\quad U' = y^{iv}$

$V''' = 0$ $\quad U^n = y^{n+2}$

$$w^n = y^{n+2} x^2 + n \cdot y^{n+1} \cdot 2x + \frac{n(n-1)}{2} \cdot y^n \cdot 2$$

$$w^n = x^2 y^{n+2} + n2x\, y^{n+1} + n(n-1) y^n$$

Let $P = xy'$

$V = x$ $\quad u = y'$

$V' = 1$ $\quad u' = y''$ $\quad r = 0$ to $r = 1$

$V'' = 0$ $\quad u^n = y^{n+1}$

$P^n = y^{n+1} \cdot x + n \cdot y^n \cdot 1$

$P^n = xy^{n+1} + ny^n$

$S = y$

$S^n = y^n$

$\therefore y^{n+2} \cdot x^2 + n \cdot 2x\, y^{n+1} + n(n-1)y^n + y^{n+1}x + ny^n + y^n = 0$

$y^{n+2}x^2 + (n \cdot 2x \cdot y^{n+1} + xy^{n+1}) + [n(n-1)y^n + ny^n + y^n] = 0$

$x^2y^{n+2} + (2n+1)x\,y^{n+1} + [n(n-1) + 1 + n]y^n = 0$

$\therefore x^2y^{n+2} + (2n+1)x\,y^{n+1} + (n^2+1)y^n = 0$