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15/ENG07/002

Petroleum Engineering

If $y = e^{x^2+x}$

$u = x^2 + x$

$du/dx = 2x + 1$

$y = e^u$

$dy/du = e^u$

$dy/dx = dy/du \times du/dx$

$= e^x * 2x + 1$

$2x + 1 e^{x}$: $u = x^2 + x$

$dy/dx = 2x + 1 e^{x^2+x}$

$d^2y/dx^2 = 2e^{x^2+x} + 1 + x^2 + 1 + x + 1e^{x^2+x}$

$y'' = d^2y/dx^2$ $y' = dy/dx$ $y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

w_1 w_2 w_3

w_1

$u = y''$ $v = 1$

$u^n = y^{n+2}$ $v = 0$

$= y^{n+2} \cdot 1 + 0$

w_2

$u = y'$ $v = 2x + 1$

$u^n = y^{n+1}$ $v' = 2$

$u^{n-1} = y^n$ $v = 0$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

W_3

$u = 2y \qquad v = 1$

$u^n = y^n \qquad v' = 0$

$= 2[(y^n \cdot 1) + 0]$

$= 2y^n$

$W_1 = W_2 + W_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2a) Using the leibnitz theorem given that $y = x^3 e^{4x}$ determine $y^{(5)}$

Solution

$u = e^{4x} \quad v = x^3$

$y^{(5)} = u^{(5)}v + 5u^4v' + 10u^3v'' + 10u^2v^3 + 5u'v^4 + u\,v^5$

$= 4^5 e^{4x} x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} 6x) + 5(4^2 e^{4x} 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 640 e^{4x} \cdot 6x + 50 e^{4x} \cdot 6$

$= 1024 e^{4x} x^3 + 3840 e^4 x^2 + 3840 e^{4x} x + 480 e^{4x}$

ii) $x^2 \dfrac{dy^2}{dx^2} + x\dfrac{dy}{dx} + y = 0$

$x^2y^2 + xy' + y = 0$

$\downarrow \qquad \downarrow \qquad \downarrow$

$w_1 \qquad w_2 \qquad w_3$

$w_1 + w_2 + w_3 = 0$

for. w_1

$u = y^t \qquad v = x^2$

$u = y^{n+2} \qquad v' = 2x$

$u^{n-1} = y^{n+1} \qquad v'' = 2$

$u^{n-2} = y^n \qquad v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \dfrac{n(n-1)y^n \cdot x}{2!} + 0$

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$= x^2 y^{(n+2)} + 2nx\,(y^{n+1}) + n(n-1)\,y^n$

for w_2

$u = y'$ $\quad u = x$

$u^n = y^{n+1}$ $\quad v = 1$

$u^{n-1} = y^n$ $\quad v^n = 0$

$= y^{n+1} \cdot x + ny^n + 0$

for w_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$= y^n \cdot 1$

$w_1 + w_2 + w_3 = 0$

$x^2 y^{n+2} + 2nxy^{n+1} + (n^2 - n)\,y^n + xy^{n+1} + ny^n + y^n$

$x^2 y^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2 y^n - ny^n + ny^n + y^n$

$x^2 y^{n+2} + 2n + 1\,(xy^{n+1}) + (n^2 + 1)\,y^n$