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CHEMICAL ENGINEERING

1. If $y = e^{x^2} + x$

$u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times 2x + 1$

but $u = x^2 + x$

$\frac{dy}{dx} = 2x + 1 e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2}$ $\quad y' = \frac{dy}{dx}$ $\quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + 1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$\underset{W_1}{y''} = \underset{W_2}{y'(2x+1)} + \underset{W_3}{2y}$

W_1

$u = y''$ $\qquad v = 1$

$u^n = y^{n+2}$ $\qquad v = 0$

$= y^{n+2} \cdot 1 + 0$

W_2

$u = y'$ $\qquad v = 2x+1$

$u^n = y^{n+1}$ $\qquad v' = 2$

$u^{n-1} = y^n$ $\qquad v = 0$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

$W_1 = W_2 + W_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

29. Using the Leibnitz theorem given that $y = x^3 e^{4x}$ determine $y^{(5)}$

SOLUTION

$u = e^{4x}$ $\qquad$ $v = x^3$

$y^{(5)} = u^{(5)}v + 5u^4 v' + 10u^3 v'' + 10u^2 v''' + 5u' v^4 + uv^5$

$= 4^5 e^{4x} . x^3 + 5(4^4 e^{4x} . 3x^2) + 10(4^3 e^{4x}(6x)) + 5(4^2 e^{4x} 6) + 0$

$= 1024e^{4x}x^3 + 1280e^{4x} . 3x^2 + 640e^{4x} . 6x + 80e^{4x} . 6$

$= 1024e^{4x}x^3 + 3840e^{4}x^2 + 3840e^{4x}x + 480e^{4x}$

ii) $x^2 \dfrac{dy^2}{dx^2} + x\dfrac{dy}{dx} + y = 0$

$$\underbrace{x^2 y''}_{w_1} + \underbrace{x y'}_{w_2} + \underbrace{y}_{w_3} = 0$$

$w_1 + w_2 + w_3 = 0$

For w_1

$u = y''$ $\qquad$ $v = x^2$

$u^n = y^{n+2}$ $\qquad$ $v' = 2x$

$u^{n-1} = y^{n+1}$ $\qquad$ $v'' = 2$

$u^{n-2} = y^n$ $\qquad$ $v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \dfrac{n(n-1)y^n . x}{2!} + 0$

$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$

For w_2

$u = y'$ $\qquad$ $u = x$

$u^n = y^{n+1}$ $\qquad$ $v = 1$

$u^{n-1} = y^n$ $\qquad$ $v'' = 0$

$= y^{n+1} . x + ny^n + 0$

~~For W_3~~

$\cancel{u = y}$ $\cancel{v = 1}$

$\cancel{u^n = y^{n+1}}$ $\cancel{v' = 1}$

$\cancel{u^{n-1} = y^n}$ $\cancel{v'' = 0}$

For W_3

$u = y$ $v = 1$

$u^n = y^n$ $v' = 0$

$= y^n \cdot 1$

$W_1 + W_2 + W_3 = 0$

$x^2 y^{n+2} + 2nxy^{n+1} + (n^2 - n) y^n + xy^{n+1} + ny^n + y^n$

$x y^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2 y^n - ny^n + ny^n + y^n$

$x^2 y^{n+2} + 2n + 1 (xy^{n+1}) + (n+1) y^n$