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### Questions

- 1)  $y = e^{x^2+x}$
- 2)  $y = x^3 e^{4x}$
- 3)  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ .

### Solution.

1)  $y = e^{x^2+x}$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$$

$$y'' = 2y + (2x+1)y'$$

hence

$$y'' = y'(2x+1) + 2y$$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + (n+1)y^n \cdot 2$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

2)  $y = x^3 e^{4x}$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} v''' + \dots$$

let  $x^3 = u$  and  $v = e^{4x}$

$$y^5 = (4)^5 e^{4x} \cdot x^3 + 5(4)^4 e^{4x} \cdot 3x^2 + 10(4)^3 e^{4x} \cdot 6x + 10(4)^2 e^{4x} \cdot 6 + 0$$

$$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x} + 0$$

$$y^5 = e^{4x} [1024x^3 + 3840x^2 + 3840x + 960]$$

3)  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ .

$$x^2 y'' + x y' + y = 0$$

$$x^2 y^{(2)} + x y^{(1)} + y = 0$$

Using Leibnitz theorem.

$$y^n = y^{(n+2)} \cdot x^2 + n \cdot 2x y^{(n+1)} + \frac{2x(n-1)y^n}{2} + y^{(n+1)} \cdot x + n y^n + y^n$$

$$y^n = x^2 y^{(n+2)} + 2x y^{(n+1)} + n(n-1)y^n + x y^{(n+1)} + n y^n + y^n$$

$$y^n = y^{(n+2)}(x^2) + y^{(n+1)}(2xn+x) + y^n(n(n-1)+n+1)$$

$$y^n = x^2 y^{n+2} + (2n+1)x y^{n+1} + (n^2-1)y^n$$

$$x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2-1)y^n = 0$$