

Q $y = e^{2x+2}$

$\frac{dy}{dx} = (2x+1)e^{2x+2}$

$= 2e^{2x+2} + (2x+1)(2x+1)e^{2x+2}$

$= e^{2x+2} [(2x+1)(2x+1)+2]$

$y' (2x+1) + 2y$

$(2x+1)e^{2x+2}(2x+1) + 2y(2x+1)e^{2x+2}$

$(2x+1) + 2e^{2x+2}$

$= e^{2x+2} [(2x+1)(2x+1)+2]$

$y'' = y' (2x+1) + 2y$

$w_1 \quad w_2 \quad w_3$

w_1

$v = y'$

$v'' = y^{(n+2)}$

w_2

$u = y$

$v = 2x+1$

$u'' = y^{(n+2)}$

$v'' = 2$

~~$u^{(n+1)} = y^{(n+1)}$~~

~~$v'' = 0 \cdot v'$~~

w_3

$u = y$

$v = 2$

$u'' = y''$

$v = 0$

$w_1 = w_2 + w_3$

$y^{(n+2)} = y^{(n+2)}(2x+1) + ny'' \cdot 2 + y''$

$= y^{(n+2)}(2x+1) + 2ny'' + y''$

$= (2x+1)y^{(n+2)} + 2(n+1)y''$

2. $y = x^3 e^{2x}$

$u = e^{2x}$

$v = x^3$

$u'' = 4^n e^{2x}$

$v^{(3)} = 3x^2$

$u^{(n+1)} = 4^{(n+1)} e^{2x}$

$v^{(n+1)} = 6x$

$u^{(n+2)} = 4^{(n+2)} e^{2x}$

$v^{(n+2)} = 6$

$u^{(n+3)} = 4^{(n+3)} e^{2x}$

$= 4^n e^{2x} x^3 + n 4^{(n-1)} e^{2x} 3x^2 + \frac{n(n-1)}{2}$

$\frac{y^{(n+1)} e^{2x} (6x + n(n-1)(n-2)4^{n-1})}{2!}$

$e^{2x} ($

$n \cdot 5$

$= 4^n e^{2x} x + 5 \cdot 4^n e^{2x} 3x^2 + 10 \cdot 4^n e^{2x} \cdot 6$

3) $2x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y$

Ass that $x^2 y^{(n+2)} + (2x+1)xy^{(n+1)}$

$+ (n^2+n)y^n = 0$

$x^2 y^{(n+2)} + xy^{(n+1)} + y = 0$

$w_1 \quad w_2 \quad w_3$

w_1

$u = y(x)$

$v = x^2$

$u^{(n+2)} = y^{(n+2)}$

$v' = 2x$

$u^{(n+1)} = y^{(n+1)}$

$v'' = 2$

$u^{(n+2)} = y^{(n+2)}$

w_2

$u = y(x^2)$

$v = x$

$u'' = y^{(n+2)}$

$v' = 1$

$u^{(n+1)} = y^{(n+1)}$

$v'' = 0$

w_3

$u = y$

$u'' = y''$

$w_1 + w_2 + w_3$

$= y^{(n+2)} x^2 + ny^{(n+1)} \cdot 2x + y''$

$\cdot x + ny'' + y'' = 0$

$x^2 y^{(n+2)} + 2xy^{(n+1)} + n(n-1)y'' + 2xy''$

$+ ny'' + y'' = 0$

$x^2 y^{(n+2)} + 2xy^{(n+1)} + ny'' + y'' + n(n-1)y''$

$+ ny'' + y''$

$= x^2 y^{(n+2)} + 2xy^{(n+1)}(2x+1) + y''(n(n-1) + n + 1)$

$= x^2 y^{(n+2)} + 2xy^{(n+1)}(2x+1) + y''$

$n(n-1) + n + 1$

$= x^2 y^{(n+2)} + 2xy^{(n+1)}(2x+1) + y''(n^2+n) = 0$