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15/ENG01/012

CHEMICAL ENGINEERING

ENG 381 ASSIGNMENT

1. If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

$y = e^{x^2+x}$

$u = e^{x^2+x}$ $\quad v = 1$

$u^n = (2x+1)^n e^{x^2+x}$ $\quad v^{(1)} = 0$

$u^{(n-1)} = (2x+1)^{n-1} e^{x^2+x}$

$y^n = u^{(n)} v + n u^{(n-1)} v^{(1)}$

$y^n = (2x+1)^n e^{x^2+x} \cdot 1 + (2x+1)^{n-1} e^{x^2+x} \cdot 0$

$y^n = (2x+1)^n e^{x^2+x}$

let $n=1$

$y^{(1)} = (2x+1)^1 e^{x^2+x}$; $u = e^{x^2+x}$; $v = 2x+1$

نبس $\quad u^{(n)} = (2x+1)^n e^{x^2+x}$ $\quad v^{(1)} = 2$

زبیب $\quad u^{(n-1)} = (2x+1)^{n-1} e^{x^2+x}$ $\quad v^{(2)} = 0$

$y'^{(n)} = (2x+1)^n e^{x^2+x} \cdot (2x+1) + n(2x+1)^{n-1} e^{x^2+x} \cdot 2$

$= (2x+1)(2x+1)^n e^{x^2+x} + 2n(2x+1)^{n-1} e^{x^2+x}$

let $n=1$

$y'' = (2x+1)(2x+1)^1 e^{x^2+x} + 2(2x+1)^0 e^{x^2+x}$

$y'' = (2x+1)(2x+1)^1 e^{x^2+2} + 2(e^{x^2+x})$

Substitute $y = e^{x^2+x}$ and $y' = (2x+1)^1 e^{x^2+x}$ in y''

$y'' = (2x+1)y' + 2y$

$y'' = y'(2x+1) + 2y$

ii) $y^{(2)} = y^{(1)}(2x+1) + 2y$

$y^2 - y^{(1)}(2x+1) - 2y = 0$

$W_1 = y^{(2)}$

$W_2 = -y^{(1)}(2x+1)$

$W_3 = -2y$

$u = y^{(2)}$ $\quad v = 1$ $\quad u = -y^{(1)}$ $\quad v = 2x+1$ $\quad u = -y$ $\quad v = 2$

$u^n = y^{(n+2)}$ $\quad v^{(1)} = 0$ $\quad u^n = -y^{(n+1)}$ $\quad v^{(1)} = 2x$ $\quad u^n = -y^n$ $\quad v' = 0$

$y^{(n-1)} = y^n$ $\quad v^{(11)} = 0$

$y(n) = 0 = u^{(n)} v + n u^{(n-1)} v^{(1)}$

$y(n) = 0 = y^{(n+2)}(1) - y^{(n+1)}(2x+1) + n(-y^n)(2x) + 2(-y^n)$

$y^n = 0 = y^{(n+2)} - (2x+1) y^{(n+1)} - 2xn(y^n) - 2(y^n)$

$y^n + 2xn(y^n) + 2(y^n) = \cancel{y^{(n+2)}} + (2x+1) y^{(n+1)} = 0\, y^{(n+2)}$

$y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^{(n)}$

2. Using the leibnitz theorem

i) $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$ $\quad v = x^3$

$u^n = 4^n e^{4x}$ $\quad v^{(1)} = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$; $v^{(2)} = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$; $v^{(3)} = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$; $v^{(4)} = 0$

$$y^{(n)} = u^{(n)} v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3$$

$$y(n) = 4^n e^{4x} \cdot x^3 + n \cdot 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2\times 1} 4^{(n-2)} e^{4x} \cdot 6x + \cdots$$

$$\cdots + \frac{n(n-1)(n-2)}{3\times 2\times 1} \cdot 4^{(n-3)} e^{4x} \cdot 6$$

$$y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^{(4)} e^{4x} \cdot 3x^2 + \frac{5(4)}{2\times 1} 4^{(3)} e^{4x} \cdot 6x + \frac{5(4)(3)}{3\times 2\times 1} \times 4^{(2)} e^{4x}$$

$\cdots + \cancel{5(n-1)(n-}$

$y^{(5)} = 1024 e^{4x} \cdot x^3 + 3840 e^{4x} \cdot x^2 + 3840 e^{4x} \cdot x + 960 e^{4x}$

$y^{(5)} = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

$y^{(5)} = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$

ii. $x^2 \, d^2y/dx^2 + x \, dy/dx + y = 0$, show that $x^2 y^{(n+2)} + (2n+1) y^{(n+1)} + (n^2+1) y^{(n)} = 0$.

also written as;

$x^2 y'' + xy' + y = 0$; $x^2 y^{(2)} + xy^{(1)} + y^{(0)} = 0$

$w_1 = x^2 y^{(2)}$ $\quad w_2 = xy^{(1)}$ $\quad w_3 = \cancel{xy}\, y^{(0)}$

$u = y^{(2)}$ $\quad v = x^2$ | $u = y^{(1)}$ $\quad v = x$ | $u = y^{(0)}$ $\quad v = 1$

$u^n = y^{(n+2)}$ $\quad v' = 2x$ | $u^n = y^{(n+1)}$ $\quad v' = 1$ | $u^n = y^n$ $\quad v^{(1)} = 0$

$u^{n-1} = y^{(n+1)}$ $\quad v'' = 2$ | $u^{n-1} = y^{(n)}$ $\quad v'' = 0$

$u^{n-2} = y^n$ $\quad v''' = 0$

$$y^n = y^{(n+2)} \cdot x^2 + n \cdot y^{(n+1)} \cdot 2x + \frac{n(n-1)}{\cancel{2} \times 1} y^n \cdot \cancel{2} + y^{(n+1)} \cdot x + n \cdot y^n \cdot 1 + y^n \cdot 1$$

$$y^n = y^{(n+2)} \cdot x^2 + 2xn\, y^{(n+1)} + n(n-1) y^{(n)} + xy^{(n+1)} + ny^n + y^n$$

$$y^n = x^2 y^{(n+2)} + 2nx\, y^{(n+1)} + (n^2 - n) y^{(n)} + xy^{(n+1)} + y^n + ny^n$$

$$y^n = x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2 - n + 1 + n)$$

$$y^n = x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2+1) = 0$$

$$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2+1) = 0.$$