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16/ENG03/062
CIVIL ENGINEERING

1) $y = e^{x^2+x}$

2) $y = x^3 e^{4x}$

3) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Solution

1) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x+4x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2y + (2x+1)y'$

hence

$y'' = y'(2x+1) + 2y$

$y^{(n+2)} = y^{(n+1)} \cdot (2x+1) + (n+1)y^n \cdot 2$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) $y = x^3 e^{4x}$

$y^n = uv + nu^{n-1}v^1 + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}v^3 + \ldots ?$

let $x^3 = v$ and $u = e^{4x}$

$y^5 = (4^0)^5 e^{4x} \cdot x^3 + 5(4)^4 e^{4x} \cdot 3x^2 + 10(4)^3 e^{4x} \cdot 6x + 10(4)^2 e^{4x} \cdot 6 + 0$

$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840xe^{4x} + 960e^{4x} + 0$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

3) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$x^2y^{(2)} + xy^{(1)} + y = 0$

Using Leibnitz theorem

$y^n = y^{(n+2)} \cdot x^2 + n \cdot 2x y^{(n+1)} + \frac{2n(n-1)y^n}{2!} + y^{(n+1)} \cdot x + n \cdot y^{(n)} \cdot 1 + y^n$

$y^n = x^2 y^{(n+2)} + 2xy^{(n+1)}n + n(n-1)y^n + xy^{(n+1)} + ny^{(n)} + y^{(n)}$

$y^n = y^{(n+2)}(x^2) + y^{(n+1)}(2xn+x) + y^{(n)}(n(n-1)+n+1)$

$y^n = x^2 y^{n+2} + (2n+1)xy^{n+1} + (n^2-1)y^n$

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-1)y^n = 0$