Agoyi Isaac Otuoze
Computer Engineering
15/ENG02/002
ENG 381 Assignment III

1) If $y = e^{x^2+x}$ Show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

$y = e^{x^2+x}$

let $u = x^2+x$

$\frac{du}{dx} = 2x+1$

$y = e^u$

$dy/du = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = e^u \times 2x+1$

where $u = x^2+x$

$\frac{dy}{dx} = e^{x^2+x} \cdot (2x+1) = y'$

If $y' = e^{x^2+x}(2x+1)$

then $y'' = 2e^{x^2} \cdot e^{x^2+x}(2) + [e^{x^2+x} \cdot (2x+1)](2x+1)$

$y'' = 2e^{x^2+x} + e^{x^2+x} \cdot (2x+1) \cdot (2x+1)$

but $y = e^{x^2+x}$

and $y' = e^{x^2+x}(2x+1)$

$y'' = 2(y) + y'(2x+1)$

Applying Lebnitz theorem to the above equation find the nth derivative

$y^{(n+2)} = 2(n+1)y^n + y^{(n+1)}(2x+1)$

2) Using Lebnitz theorem given that

$y = x^3 e^{4x}$ determine $y^{(5)}$

Solution

Recall that lebnitz theorem states that

$y^5 = u^5 v + 5u^4 v' + 10u^3 v^2 + 10u^2 v^3 + 5u' v^4 + v^5$

where $u = e^{4x}$

$u' = 4e^{4x}$ $\quad$ $v = x^3$

$u^2 = 16e^{4x}$ $\quad$ $v' = 3x^2$

$u^3 = 64e^{4x}$ $\quad$ $v^2 = 6x$

$u^4 = 256e^{4x}$ $\quad$ $v^3 = 6$

$u^5 = 1024e^{4x}$

Therefore

$y^5 = 1024e^{4x}(x^3) + 5(256e^{4x})(3x^2) + 10(64e^{4x})(6x) + 10(16e^{4x})$
$\times 6 + 5(4e^{4x})(0) + 0^5$

$= 1024x^3 \cdot e^{4x} + 3840x^2 e^{4x} + 3840xe^{4x} + 960e^{4x}$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

ii) If $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that ~~$x^2 (y$~~ $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Solution

The equation can be written as $x^2 y'' + xy' + y = 0$

Let $W_1 = x^2 y''$, $W_2 = xy'$ and $W_3 = y$

Solving for $W_1 = x^2 y''$

Let $u = y^2$, $u^n = y^{n+2}$

Let $v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$W_1 = y^{(n+2)} x^2 + ny^{(n+1)} 2x + \frac{n(n-1)}{2!} y^n \times 2 + 0$

$W_2 = xy'$

let $u = y'$; $u^n = y^n$

let $v = x$, $v' = 1$ and $v'' = 0$

$W_2 = y^{(n+1)} \cdot x + ny^n \times 1 + 0$

$= xy^{n+1} + ny^n$

Scanned by CamScanner

$W_3 = y^{(n)}$

Combining

$W = W_1 + W_2 + W_3$

$W = x^2 y^{(n+2)} + n y^{(n+1)} \cdot 2x + \frac{n(n-1)}{2} y^{n+2} + x y' + y^{(n+1)} x$

$+ n y^n + y^n = 0$

$x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^n (n(n-1) + n+1)] = 0$

$x^2 y^{(n+2)} + (2n+1) x y^{n+1} + (n^2+1) y^n = 0$