

$$1) \frac{d^2y}{dx^2} - 2y = 8 \text{ Inhomogeneous form, } \frac{d^2y}{dx^2} - 2y = 0$$

$$m^2 - m - 2 = 0 \Rightarrow \text{auxiliary form}$$

$$m^2 - 2m - 2 = 0$$

$$m(m-2) + (m-2) = 0$$

$$(m+1)(m-2) = 0$$

$$m+1=0 \quad ; \quad m-2=0$$

$$m = -1$$

$$m_2 = 2$$

$$CF = y = Ae^{-x} + Be^{2x}$$

$$PI = y = c$$

$$\frac{dy}{dx} = 0 \quad \frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2(c) = 8$$

$$-2c = 8$$

$$c = 8/2 \quad c = -4$$

$$G.S = CF + PI$$

$$\therefore G.S = y = Ae^{-x} + Be^{2x} - 4$$

$$2) \frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

$$\text{homogeneous form} = \frac{d^2y}{dx^2} - 4y = 0$$

case 5:

$$\text{In auxiliary form } m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \sqrt{4}$$

$$m = +2$$

$$CF = y = C \cosh 2x + D \sinh 2x$$

$$y = C \cosh 2x + D \sinh 2x$$

$$P.I = y = Ce^{3x} \frac{dy}{dx} = 3Ce^{3x} \frac{dy}{dx} = 9Ce^{3x}$$

$$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$5C = 10 \quad C = 10/5 \quad C = 2$$

$$G.S = CF + P.I$$

$$G.S = y = C \cosh 2x + D \sinh 2x + 2e^{3x}$$

$$3) \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

In homogenous form

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

In auxiliary form

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0, m = -1$$

$$f = e^{-2x} (A + Bx)$$

$$P.I = y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$7Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} = e^{-2x}$$

$$C = \frac{e^{-2x}}{e^{-2x}} \quad C = 1$$

$$G.S = e^{-2x} (A + Bx) + e^{-2x}$$

$$4) \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

In homogenous form

$$m^2 + 25 = 0, m^2 = -25, m = \pm 5j$$

$$m = \sqrt{-1} \times \sqrt{25}; m = \pm 5j$$

$$C.F. y = C \cos 5x + D \sin 5x$$

$$P.I y = Ax^2 + Bx + C$$

$$\frac{dy}{dx} = 2Ax + B; \frac{d^2y}{dx^2} = 2A$$

$$2A + 25(Ax^2 + Bx + C) = 5x^2 + x$$

$$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$$

$$2C + 25C + 25Dx + 25Ax^2 = 5x^2 + x$$

$$2C + 25C = 0$$

$$27C = 0 \quad C = 0$$

$$25D = 1 \quad D = 1/25$$

$$25A = 5 \quad A = 1/5$$

$$2C + 25E = 0$$

$$2(1/5) + 25E = 0$$

$$25E = -2/5; E = -2/5 \div 25$$

$$E = -2/5 \times 1/25$$

$$E = -2/125$$

$$P.I = y = 1/5x^2 + 1/25x - 2/125$$

$$G.S = (C \cos 5x + D \sin 5x + 1/5x^2 + 1/25x - 2/125)$$

$$5) \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4 \sin x$$

In homogenous form; $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

In auxiliary form; $m^2 + 2m + 1 = 0$

$$m^2 - m - m + 1 = 0$$

$$m(m-1)(m-1) = 0$$

$$(m-1)(m-1) = 0 \quad m = 1$$

$$C.F. = y = e^x (A + Bx)$$

$$P.I = y = A \cos x + B \sin x$$

$$\frac{dy}{dx} = -A \sin x + B \cos x$$

$$\frac{d^2y}{dx^2} = -A \cos x - B \sin x$$

$$-A \cos x - B \sin x = (-A \sin x + B \cos x)$$

$$+ A \cos x + B \sin x = 4 \sin x$$

$$-A \cos x - B \sin x + 2A \sin x - 2B \cos x$$

$$+ A \cos x + B \sin x = 4 \sin x$$

$$2A \sin x - 2B \cos x = 4 \sin x$$

$$2A = 4$$

$$= \frac{4}{2} = 2$$

$$2B = 0 \Rightarrow B = \frac{0}{2} = 0$$

$$P \cdot I = y = 2 \cos x$$

$$G.S: y = e^{2x}(A + Bx) + 2 \cos x$$

$$\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 5y = 2e^{-2x}$$

$$\frac{dy}{dx} + 4 \frac{dy}{dx} + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$m = \frac{-4 \pm \sqrt{4}}{2}$$

$$m = \frac{-4 + \sqrt{-1} \times \sqrt{4}}{2}$$

$$m = \frac{-4 + j2}{2}$$

$$m = -2 + j$$

$$\beta = 1, \alpha = -2$$

$$y = e^{-2x}(A \cos x + B \sin x)$$

$$P.I: y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2 y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$$

$$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$$

$$4C - 8C + 5C = 2; C = 2$$

$$P.I: y = 2e^{-2x}$$

$$G.S: y = e^{2x}(A \cos x + B \sin x) + 2e^{-2x}$$

$$\frac{dy}{dx} = Vdu + Udv; u = e^{-2x}; v = A \cos x + B \sin x$$

$$\frac{dy}{dx} = -2e^{-2x}(A \cos x + B \sin x) + e^{-2x}(-A \sin x + B \cos x) - 4e^{-2x}$$

$$\text{Since } \frac{dy}{dx} = -2, x = 0, y = 1$$

$$1 = e^{-2(0)}(A \cos 0 + B \sin 0) + 2e^{-2(0)}$$

$$1 = 1(A + 0) + 2; 1 = A + 2; A = -1$$

$$A = -1$$

$$-2 = -2e^{-2(0)}(A \cos 0 + B \sin 0) + e^{-2(0)}(-A \sin 0 + B \cos 0) - 4e^{-2(0)}$$

$$(A - \sin 0 + B \cos 0) - 4e^{-2(0)}$$

$$-2 = -2(A + 0) + 1(0 + B) - 4$$

$$-2 = -2A + B - 4; -2A + B = 2$$

$$-2(-1) + B = 2; 2 + B = 2; B = 0$$

$$B = 0$$

$$P.S: y = e^{2x}(-\cos x + B \sin x) + 2e^{-2x}$$

$$y = e^{-2x}(-\cos x + 0 \sin x) + 2e^{-2x}$$

$$y = e^{-2x}(-\cos x) + 2e^{-2x}$$

$$y = e^{-2x}(2 - \cos x)$$

$$3 \frac{d^2 y}{dx^2} - \frac{dy}{dx} - y = 2x - 3$$

$$3m^2 - 2m - 1 = 0$$

$$3m^2 - 3m + m - 1 = 0$$

$$3m(m-1) + 1(m-1) = 0$$

$$(m-1)(3m+1) = 0; m_1 = 1; m_2 = -\frac{1}{3}$$

$$y = Ae^x + Be^{-\frac{1}{3}x}$$

$$y = Cx + D$$

$$\frac{dy}{dx} = C; \frac{d^2 y}{dx^2} = 0$$

$$\frac{dy}{dx} = C; \frac{d^2 y}{dx^2} = 0$$

$$3(0) - 2(C) - (Cx + D) = 2x - 3$$

$$-2C - Cx - D = 2x - 3$$

$$-2C - D - Cx = 2x - 3$$

$$-C = 2; C = -2$$

$$-2C - D = -3$$

$$4 - 7 = -3$$

$$4 + 3 = 7$$

$$p = -7$$

$$PI = 2x + 3$$

$$G.S = Ae^{2x} + Be^{-2x} / 3 - 2x + 7$$

$$\int \frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 8y = 8e^{4x}$$
$$\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 8y = 8e^{4x}$$

$$\frac{d^2y}{dx^2}$$

$$m^2 - 6m + 8 = 0$$

$$m^2 - 4m - 2m + 8 = 0$$

$$m(m-4) - 2(m-4) = 0$$

$$(m-4)(m-2) = 0$$

$$m-4=0; m-2=0$$

$$m_1 = 4; m_2 = 2$$

$$C.F = y = Ae^{4x} + Be^{2x}; P.I = Cx e^{4x}$$

$$y = UV; U = Cx, V = e^{4x}$$

$$\frac{dy}{dx} = Udu + Udv; du = C, dv = 4e^{4x}$$

$$\frac{dy}{dx} = e^{4x} + Cx \cdot 4e^{4x} = Ce^{4x} + 4Cx e^{4x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{4x} + 4Ce^{4x} + 16Cx e^{4x}$$
$$= 8Ce^{4x} + 16Cx e^{4x}$$

$$8Ce^{4x} + 16Cx e^{4x} - 6(Ce^{4x} + 4Cx e^{4x}) + 8(Cx e^{4x}) = 8e^{4x}$$

$$8C + 16Cx - 6C - 24Cx + 8Cx = 0$$

$$8C - 6C = 8$$

$$2C = 8; C = 8/2; C = 4$$

$$P.I = y = 4x e^{4x}$$

$$G.S = C.F + P.I$$

$$y = Ae^{4x} + Be^{2x} + 4x e^{4x}$$