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1. IF $y = e^{x^2+x}$

show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

$y = e^{x^2+x}$ ... (1)

Taking the first derivative

$dy/dx = (2x+1)e^{x^2+x} \Rightarrow y'$ ... (2)

$d^2y/dx^2 = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}$ (2)

$d^2y/dx^2 = (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$ ... (3)

Equating (3) to (2) and (1)

$(4x^2+4x+1)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$(4x^2+4x+1)e^{x^2+x} = (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$ ... (4)

$\therefore y'' = y'(2x+1) + 2y$

1(ii) $y'' = y'(2x+1) + 2y$

Using Leibnitz Theorem

$y'' + y'(2x+1) + 2y = 0$

$y'' \Rightarrow d^2y/dx^2$

$u = y''$, $u^n = y^{(n+2)}$

$v = 1$, $v' = 0$

$y^{(n)} = y^{(n+2)} + 0 \Rightarrow y^{(n+2)}$ ... (1)

$u = y''$, $u^{(n)} = y^{(n+1)}$

$v = 2x+1$, $v' = 2$, $v'' = 0$

$y^{(n)} = y^{(n+1)}2x + 1 + ny^{(n)}2 + 0$ ... (2)

$u = y$, $u^n = y^{(n)}$

$v = 2$, $v' = 0$

$y^{(n)} = y^{(n)}2 + 0 = 2y^{(n)}$ ... (3)

Combining the three equations

$y^{(n+2)} = y^{(n+1)}2x + 1 + 2ny^{(n)} + 2y^{(n)}$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{(n)}(n+1)$

2.) Using the leibnitz theorem given that

i) $y = x^3 e^{4x}$, determine $y^{(5)}$

solution

$y' = x^3(4e^{4x}) + e^{4x}(3x^2)$

$y' = x^3 4e^{4x} + 3x^2 e^{4x}$ ... (1)

$y'' = x^3(16e^{4x}) + 4e^{4x} 3x^2 + 3x^2 4e^{4x} + 6xe^{4x}$

$y''' = x^3 64e^{4x} + 16e^{4x} 3x^2 + 3x^2 16e^{4x} + 6x4e^{4x} + 3x^2 16e^{4x} + 6x4e^{4x} + 6x^2 4e^{4x} + 6e^{4x}$

$y^{iv} = x^3 256e^{4x} + 3x^2 64e^{4x} + 3x^2 64e^{4x} + 6x16e^{4x} + 3x^2 64e^{4x} + 3x^2 64e^{4x} + 6x16e^{4x} + 96e^{4x} + 3x^2 64e^{4x} + 6x16e^{4x} + 24e^{4x} + 6x16e^{4x} + 24e^{4x} + 24e^{4x} + 0$

$y^v = x^3 1024e^{4x} + 3x^2 256e^{4x} + 3x^2 256e^{4x} + 6x64e^{4x} + 3x^2 256e^{4x} + 6x64e^{4x} + 6x64e^{4x} + 96e^{4x} + 3x^2 256e^{4x} + 6x64e^{4x} + 96e^{4x} + 384e^{4x} + 0 + 3x^2 256e^{4x} + 6x64e^{4x} + 6x64e^{4x} + 96e^{4x} + 6x64e^{4x} + 96e^{4x} + 0 + 6x64e^{4x} + 96e^{4x} + 96e^{4x} + 0 + 96e^{4x} + 0$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

solution

$w = x^2 y''$

$V = x^2, V' = 2x, V^2 = 2, V^3 = 0$

$U = y'', U^n = y^{(n+2)}$

$y^{(n)} = y^{(n+2)}x^2 + ny^{(n+2-1)} \cdot 2x + \frac{n(n-1)}{2} y^{(n+2-2)} 2 + 0$

$= y^{(n+2)}x^2 + 2nxy^{(n-1)} + n(n-1)y^{(n)}$

$w = xy'$

$V = x, V' = 1, V^2 = 0$

$w^{(n)} = y^{(n+1)}x + ny^{(n+1-1)} + 0$

$= y^{(n+1)}x + ny^n$

$w = y$

$V = 1, V' = 0$

$u = y \quad u^{(n)} = y^{(n)}$

$(u)^{(n)} = y^n$

Then $(x^2y'' + xy' + y)^n = 0$ becomes

$y^{(n)} = x^2 y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^{(n)} + ny^{(n)} + y^{(n)}$

$y^{(n)} = x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-n)y^{(n)} + ny^{(n)} + y^{(n)}$

$y^{(n)} = x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-n+n+1)y^{(n)}$

$\Rightarrow x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$