1) If $y = e^{x^2+x}$

show that $y'' = y'(2x+1) + 2y$

and hence, prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

<u>Solution</u>

$y = e^{x^2+x}$

$y' = (2x-1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2y + (2x+1)y'$

$y'' = y'(2x+1) + 2y$

$y^{(n+2)} = y^{(n+1)} \cdot (2x+1) + (n+1)y^n \cdot 2$

$\therefore y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$

2) Using the Leibnitz theorem, given that

i) $y = x^3 e^{4x}$, determine y^5

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that

$$x^2 y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$$

<u>Solution</u>

i) $y = x^3 e^{4x}$

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \ldots$

Let $x^3 = v$ and $e^{4x} = u$

$y^5 = 4^5 \cdot e^{4x} \cdot x^3 + 5 \cdot 4^4 \cdot e^{4x} \cdot 3x^2 + 10 \cdot 4^3 \cdot e^{4x} \cdot 6x + 10 \cdot 4^2 \cdot e^{4x} \cdot 6 + 0$

$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x} + 0$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y^0 = 0$

$x^2y^2 + xy' + y^0 = 0$

Using Leibnitz theorem;

$y^n = y^{n+2}\cdot x^2 + n\cdot 2xy^{n+1} + \frac{2n(n-1)}{2!}y^n + y^{n+1}\cdot x$

$+ n\cdot y^n\cdot 1 + y^n$

$y^n = x^2y(n+2) + 2xy^{(n+1)}\cdot n + n(n-1)y^n + xy^{n+1}$

$+ ny^n + y^n$

$y^n = y^{n+2}\cdot x^2 + y^{n+1}(2xn + x) + y^n(n(n-1) + n + 1)$

$y^n = x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2 - n + n + 1)y^n$

$y^n = x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2 - 1)y^n$

$\therefore x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2-1)y^n = 0$