

NAME : RICHARD-KOKO OKOYTE
 MATRIC NO : 151ENG011015
 DEPARTMENT : CHEMICAL ENGINEERING
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1 $y = e^{x^2+x}$

$y' = \frac{dy}{dx} = 2x+1 (e^{x^2+x})$

$y'' = \frac{d^2y}{dx^2} = e^{x^2+x} \cdot (2) + (2x+1) (2x+1 (e^{x^2+x}))$

$y'' = 2y + (2x+1)y'$

$y'' = \underbrace{y'}_{w_1} (2x+1) + \underbrace{2y}_{w_2}$

For w_1

$u = y^2 \quad v = 1$

$u^n = y^{(n+2)}$

For w_2

$u = y' \quad v = 2x+1$

$u^n = y^{(n+1)} \quad v' = 2$

$u^{n-1} = y^n$

For w_3

$u = 2y \quad v = 2$

$u^n = y^n$

$y^n = u^n v + n u^{n-1} v'$

$y^{n+2} = (2x+1)y^{(n+1)} + n y^{(n)} \cdot 2 + y^n \cdot 2$

$y^{n+2} = (2x+1)y^{n+1} + 2n y^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

2 $y = x^3 e^{4x}$

SOLN

$u = e^{4x}$

$v = x^3$

$u^n = 4^n e^{4x}$

$v' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$

$v'' = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$

$v''' = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$

$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v'''$

$y^5 = 4^5 e^{4x} \cdot x^3 + 5(4^{(5-1)} e^{4x}) \cdot 3x^2 + \frac{5(5-1)}{2!} (4^{(5-2)} e^{4x}) \cdot 6x + \frac{5(5-1)(5-2)}{3!} (4^{(5-3)} e^{4x}) \cdot 6$

$y^5 = 1024 e^{4x} \cdot x^3 + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

$y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

$y^5 = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$

2.ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

SOLN

$\underbrace{x^2 y^2}_{w_1} + \underbrace{x y^1}_{w_2} + \underbrace{y^0}_{w_3} = 0$

For w_1

$u = y^2$

$v = x^2$

$u^n = y^{(n+2)}$

$v' = 2x$

$u^{(n-1)} = y^{(n+1)}$

$v'' = 2$

$u^{(n-2)} = y^n$

For w_2

$u = y^1$

$v = x$

$u^n = y^{(n+1)}$

$v' = 1$

$u^{(n-1)} = y^n$

For w_3

$u = y$

$v = 1$

$u^n = y^n$

$$y^n = u^n + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \dots$$

$$x^2 y^{(n+2)} + 2xy^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + 2xy^{(n+1)} + n(n-1)y^{(n)} + ny^{(n)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n(n-1) + n+1)y^{(n)} = 0$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2 - n + n + 1)y^{(n)} = 0$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2 + 1)y^{(n)} = 0$$