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Computer Engineering

① (i) If $y = e^{x^2+x}$
show that
$y'' = y'(2x+1) + 2y$
and hence prove that
$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2(n+1)y^{n}$

Solution

$y = e^{x^2+x}$ —— (1)

taking the first derivative

$\frac{dy}{dx} = (2x+1)\,e^{x^2+x}$ as y' —— eqn (2)

taking the second derivative using product rule

$\frac{d^2y}{dx^2} = (2x+1)(2x+1)\,e^{x^2+x} + e^{x^2+x}(2)$

$\frac{d^2y}{dx^2} = (4x^2+4x+1)\,e^{x^2+x} + 2e^{x^2+x}$ —— eqn(3)
$\rightarrow y''$

Now equating eqn(3) to eqn (2) and eqn(1)

$(4x^2+4x+1)\,e^{x^2+x} = (2x+1)\,e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$(4x^2+4x+1)\,e^{x^2+x} = (4x^2+4x+1)\,e^{x^2+x} + 2e^{x^2+x}$ —— (4)

from eqn (4)

Hence: $y'' = y'(2x+1) + 2y$ //

(ii) $y'' = y'(2x+1) + 2y$

using Leibnitz theorem

$-y'' + y'(2x+1) + 2y = 0$

Taking the first term

$y'' \Rightarrow d^2y/dx^2$

$u = y''$, $u^n = y^{(n+2)}$

$v = 1$, $v' = 0$

$y^{(n)} = y^{(n+2)} + 0 \Rightarrow y^{(n+2)} \longrightarrow$ (i)

taking the second term

$u = y'$ $u^n = y^{(n+1)}$

$v = 2x+1$ $v' = 2$ $v'' = 0$

$y^{(n)} = y^{(n+1)} 2x+1' + n y^{(n)} 2 + 0$ —— (ii)

taking the third term

$u = y$ $u^n = y^{(n)}$

$v = 2$, $v' = 0$

$y^{(n)} = y^{(n)} 2 + 0 \Rightarrow 2y^{(n)}$ —— (iii)

Combining the three equations

$y^{(n+2)} = y^{(n+1)} 2x+1 + 2n y^{(n)} + 2y^{(n)}$

Collect like terms

$y^{(n+2)} = 2x+1\, y^{(n+1)} + 2y^{(n)} (n+1)$ //

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(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

solution

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

taking the first term

$x^2 \frac{d^2y}{dx^2} \Rightarrow x^2 y''$

$u = y''$, $u^n = y^{(n+2)}$

$v = x^2$ $v' = 2x$, $v'' = 2$, $v''' = 0$

$$y^n = y^{(n+2)} x^2 + n y^{(n+2-1)} 2x + \frac{n(n-1)}{\cancel{2}!} y^{(n+\cancel{2}-\cancel{2})} \cancel{2} + 0$$

$$y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)} \text{ — } (1)$$

taking the second term :

$x\frac{dy}{dx} \Rightarrow xy'$

$u = y'$, $u^n = y^{n+1}$

$v = x$, $v' = 1$, $v'' = 0$

$$y^n = y^{(n+1)}x + n y^{(n+1-1)} + 0$$

$$y^n = x y^{(n+1)} + n y^{(n)} \text{ ——— } (2)$$

taking the third term :

$u = y$, $u^n = y^n$

$v = 1$, $v' = 0$

$y^n = y^n + 0 \Rightarrow y^n$ ——— (3)

Now combining eqn (1) (2) & (3)

$y \cdot x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)}$

$+ y^n = 0$

Now collecting like terms.

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + y^{(n)} (n^2 - \cancel{n} + \cancel{n} + 1) =$

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + y^{(n)} (n^2 + 1) = 0$

2 (i)

$y = x^3 e^{4x}$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \cdots$

let $x^3 = v$ and $e^{4x} = u$

$y^5 = 4^5 e^{4x} x^3 + 5 \cdot 4^4 e^{4x} 3x^2 + 10 \cdot 4^3 e^{4x} 6x$
$+ 10 \cdot 4^2 \cdot e^{4x} \cdot 6 + 0$

$y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x}$
$+ 960 e^{4x} + 0$

$y^5 = e^{4x} (1024 x^3 + 3840 x^2 + 3840 x + 960)$ //