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JIENG04P1042

ELECT/ELECT ENGR.

1) If $y = e^{x^2+x}$

$u = x^2 + x$

$du/dx = 2x + 1$

$y = e^u$

$dy/du = e^u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$= e^u \times 2x+1$

$2x+1\, e^u$ $\qquad u = x^2 + x$

$$\frac{dy}{dx} = 2x+1\, e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$$

$= 2e^{x^2+x} + 4x^2 + 4x + 1\, e^{x^2} + x$

$$\frac{d^2y}{dx^2} = 2ex^{2+x} + 4x^2 + 4x + 1\, e^{x^2} + x$$

$y'' = \frac{d^2y}{dx^2}$ $\qquad y' = \frac{dy}{dx}$ $\qquad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' - 2e^{x^2+x} + 4x^2 + 4x + 1\, e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$
$= 4x^2 + 4x + 1 e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1 e^{x^2+x}$
$= 2e^{x^2+x} + 4x^2 + 4x + 1 e^{x^2+x}$

$\underset{w_1}{y''} \quad \underset{w_2}{y'(2x+1)} + \underset{w_3}{2y}$

w_1

$u = y''$ $\quad v = 1$
$d\ u^n = y^{n+2}$ $\quad v = 0$
$= y^{n+2} \cdot 1 + 0$

$d\ w_2$

$u = y'$ $\quad v = 2x+1$
$u^n = y^{n+1}$ $\quad v' = 2$
$u^{n-1} = y^n$ $\quad v = 0$
$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$
$= y^{n+1}(2x+1) + 2n(y^n)$

w_3

$u = 2y$ $\quad v = 1$
$u^n = y^n$ $\quad v' = 0$

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$= 2[(y^n . 1) + 0]$

$= 2y^n$

$w_1 = w_2 + w_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2a) using the leibnitz theorem given that $y = x^3 e^{4x}$ determine $y^{(5)}$

Solution

$u = e^{4x}$ $v = x^3$

$y^{(5)} = u^{(5)}v + 5u^4v' + 10u^3v'' + 10^2u^2v^3 + 5u'v^4 + uv^5$

$= 4^5e^{4x} . x^3 + 5(4^4e^{4x} . 3x^2) + 10(4^3e^{4x} 6x) + 15(4^2e^{4x} . 6)$

$+ 0$

$= 1024e^{4x}x^3 + 1280e^{4x} . 3x^2 + 640e^{4x} . 6x + 80e^{4x} . 6$

$= 1024e^{4x}x^3 + 3840e^4x^2 + 3840e^{4x}x + 480e^{4x}$

2) $x^2 \frac{dy^2}{dx^2} + x\frac{dy}{dx} + y = 0$

$\underset{w_1}{x^2y''} + \underset{w_2}{xy'} + \underset{w_3}{y} = 0$

$w_1 + w_2 + w_3 = 0$

For w_1

$u = y''$ $\quad v = x^2$

$u^n = y^{n+2}$ $\quad v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$u^{n-2} = y^n$ $\quad v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})\, 2x + \frac{n(n-1)}{2!} y^n \cdot \cancel{2} + 0$

$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$

For w_2

$u = y'$ $\quad v = x$

$u^n = y^{n+1}$ $\quad v' = 1$

$u^{n-1} = y^n$ $\quad v'' = 0$

$= y^{n+1} \cdot x + ny^n + 0$

For w_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$= y^n \cdot 1$

$w_1 + w_2 + w_3 = 0$

$x^2 y^{n+2} + 2nxy^{n+1} + (n^2 - n)y^n + xy^{n+1} + ny^n + y^n$

$x^2 y^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2 y^n - \cancel{ny^n} + \cancel{ny^n} + y$

$x^2 y^{n+2} + (2n+1)(xy^{n+1}) + (n^2+1)y^n$

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