15/ENG02/005
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(i) $y = e^{x^2} + x$ Show that $y'' = y'(2x+1) + 2y$

$y'' = y'(2x+1) + 2y$

$y'' = \frac{d^2y}{dx^2}$ $y' = \frac{dy}{dx}$

$\frac{dy}{dx} = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1)e^{x^2+x}$

$= (4x^2+4x+3)e^{x^2+2x}$

Sub RHS & LHS

$(4x^2 + 4x+3)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$= (2x^2+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1+3)e^{x^2+x}$

$= (4x^2+4x+4)e^{x^2+x}$

$(4x^2+4x+3)e^{x^2+x} = (4x^2+4x+3)e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$ //

(ii) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

let $w = y''$

$v = 1$ $v' = 0$

$u = y''$ $u^n = y^{n+2}$

$w^n = u^n v + n u^{n-1} v'$

$= y^{(n+2)} + 0$

let $w = -y'(2x+1)$

$v = 2x+1$ $v' = 2$ $v'' = 0$

$u = -y'$ $u^n = -y^{n+1}$

$w^n = u^n v + n u^{n-1} v^1 + n(n-1) u^{n-2} v^{11}$

$= y^{n+1}(2x+1) + n(-y^{n+1-})(2) + 0$

$= -y^{n+1}(2x+1) + 2n(-y^n)$

let $w = -2y$

$v = -2 \quad v^1 = 0$

$u = y \quad u^n = y^n$

$w^n = u^n v + n u^{n-1} v^1$

$= y^n - 2 + 0$

$= -2y^n$

$\therefore y^n = y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^2$

$y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^n(n-1)$

2(i) $y = x^3 e^{4x}$ find y^5

$v = x^3, \quad v^1 = 3x^2, \quad v^{11} = 6x, \quad v^{111} = 6, \quad v^4 = 0$

$u = e^{4x}, \quad u^1 = 4e^{4x}, \quad u^{11} = 16e^{4x}, \quad u^{111} = 64e^{4x},$

$u^4 = 256e^{4x}, \quad u^5 = 1024e^{4x}$

$y^n = u^n v + n v^{n-1} v^1 + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} u^3$

$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^4$

$y^5 = 1024 e^{4x}(x^3) + 15x(256e^{4x}) + 60x(64e^{4x})$

$+ 60(16e^{4x})$

$y^5 = x^3 1024 e^{4x} + x^2 3840 e^{4x} + x 3840 e^{4x} + 960 e^{4x}$

$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$ //

2(ii) $x^2 dy/dx + x\, dy/dx + y = 0$

$x^2 y^{11} + xy + y = 0$

let $w = x^2 y^{11}$

$v = x^2, \quad v^1 = 2x, \quad v^{11} = 2, \quad v^{111} = 0$

$u = y''$ $\quad u^n = y^{n+2}$

$w^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-2} v'''$

$= y^{n+2}(x^2) + n(y^{n+2})(2x) + \frac{n(n-1)}{2!}(y^{n+2-2})(2)$

$-x^2 y^{n+2} + 2xn(y^{n+1}) + n(n-1)(y^n)$

let $w = xy$

$v = x \quad , \quad v' = 1 \quad , \quad v'' = 0$

$u = y' \quad , \quad u^n = y^{n+1}$

$w^n = y^{n+1}(x) + n(y^{n+1-1})(1) + 1$

$= xy^{n+1} + ny^n$

let $w = y$

$v = 1 \quad v' = 0$

$u = y \quad u^n = y^n$

$w^n = y^n$

$y^n = x^2 y^{(n+2)} + 2xn(y^{n+1}) + n(n-1)(y^n) + xy^{n+1}$
$+ ny^n + y$

$x^2 y(n+2) + (2n+1)x^{(n+1)} + (n^2 - n + n + 1) y^n = 0$

$x^2 y^{n+2} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$