MUTU EMMANUELLA TOMBRAPADE

15/ENG07/029

PETROLEUM ENGINEERING

ENG 381

1 If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ and hence, prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

$y = e^{x^2+x}$; $u = e^{x^2+x}$ $\quad v = 1$

$u^n = (2x+1)^n e^{x^2+x}$ $\quad v' = 0$

$u^{n-1} = (2x+1)^{n-1} e^{x^2+x}$

$y^{(n)} = u^n v^0 + nu^{(n-1)} v^{(1)}$; $y^n = (2x+1)^n e^{x^2+x}$

let $n = 1$

$y' = (2x+1)' e^{x^2+x}$; $u = e^{x^2+x}$; $v = 2x+1$

$u^n = (2x+1)^n e^{x^2+x}$ $\quad v^{(1)} = 2$

$u^{n-1} = (2x+1)^{n-1} e^{x^2+x}$ $\quad v'' = 0$

$y^{1(n)} = (2x+1)^n e^{x^2+x} \cdot (2x+1) + n(2x+1)^{n-1} e^{x^2+x} \cdot 2$

$= (2x+1)(2x+1)^n e^{x^2+x} + 2n(2x+1)^{n-1} e^{x^2+x}$

let $n = 1$

$y'' = (2x+1)(2x+1)' e^{x^2+x} + 2(2x+1)^0 e^{x^2+x}$

$y'' = (2x+1)(2x+1)' e^{x^2+x} + 2(e^{x^2+x})$

Subst. $y = e^{x^2+x}$ and $y' = (2x+1)' e^{x^2+x}$ in y''

$y'' = (2x+1)y' + 2y$

$y'' = y'(2x+1) + 2y$

$u = y$, $v = 2x+1$	; $u = y$, $v = 2$	
$u^n = y^{n+1}$	$v' = 2$	$u^n = y^n$, $v' = 0$
$u^{n-1} = y^n$	$v'' = 0$	

This can be written as

$y^{(2)} = y^{(1)}(2x+1) 2y$

$y^{(n)} = u^{(n)} v + nu^{(n-1)} v^{(1)}$

$y^{(n+2)} = y^{(n+1)}(2x+1) + \cdots + ny^{(n)} \cdot 2 + y^n \cdot 2$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

2 Using the Leibnitz theorem given that

i $y = x^3 e^{4x}$, determine y^5.

$u = e^{4x}$ $\quad v = x^3$

$u^n = 4^n e^{4x}$ $\quad v' = 3x^2$

$u^{n-1} = 4^{(n-1)} e^{4x}$ $\qquad v'' = 6x$

$u^{n-2} = 4^{(n-2)} e^{4x}$ $\qquad v''' = 6$

$u^{n-3} = 4^{(n-3)} e^{4x}$ $\qquad v^{iv} = 0$

$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)u^{(n-2)}}{2!} v'' + \frac{n(n-1)(n-2)u^{(n-3)}}{3!} v''' \cancel{+ n(n-1)(n-2)(n-3)u^{n-4}}$

$y^n = 4^n e^{4x} \cdot x^3 + n4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1)\cdot 4^{(n-2)}}{\cancel{2}} e^{4x} \cdot \cancel{6x}^{3} + \frac{n(n-1)(n-2)\cdot 4^{(n-3)}}{\cancel{3\times 2}} e^{4x} \cdot \cancel{6}$

$y^n = 4^n e^{4x} \cdot x^3 + n4^{(n-1)} e^{4x}\, 3x^2 + n(n-1)\cdot 4^{(n-2)} e^{4x} \cdot 3x + n(n-1)(n-2)\cdot 4^{(n-3)} e^{4x}.$

$y^5 = \cancel{1024x^3e^{4x}}\ 4^5 e^{4x} \cdot x^3 + 5\cdot 4^{(5-1)} e^{4x} \cdot 3x^2 + 5(5-1)\cdot 4^{(5-2)} e^{4x} \cdot 3x + 5(5-1)(5-2)\cdot 4^{(5-3)} e^{4x}$

$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x}$

$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$

ii $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

$= x^2 y'' + xy' + y = 0 \Rightarrow x^2 y^{(2)} + xy^{(1)} + y^{(0)} = 0$

$u = y''$	$v = x^2$	$u = y'$	$v = x$	$u = y$	$v = 1$
$u^n = y^{(n+2)}$	$v' = 2x$	$u^n = y^{n+1}$	$v' = 1$	$u^n = y^n$	$v' = 0$
$u^{(n-1)} = y^{(n+1)}$	$v'' = 2$	$u^{n-1} = y^n$	$v'' = 0$		
$u^{(n-2)} = y^n$	$v''' = 0$				

$y^n = u^n v + nu^{(n-1)} v' + \frac{n(n-1)u^{(n-2)}}{2!} v''$

$y^n = y^{n+2} x^2 + ny^{(n+1)} \cdot 2x + \frac{n(n-1)y^n}{\cancel{2}} \cdot \cancel{2} + y^{n+1} \cdot x + ny^n \cdot 1 + y^n \cdot 1$

$y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n$

$y^n = x^2 y^{(n+2)} + 2nxy^{(n+1)} + xy^{(n+1)} + (n^2-n)y^n + ny^n + y^n$

$y^n = x^2 y^{n+2} + (2n+1)xy^{(n+1)} + (n^2 - \cancel{n} + \cancel{n} + 1)y^n$

$y^n = x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n$

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$ //