NAME: [illegible]
DEPARTMENT: CHEMICAL ENGINEERING
MATRIC NO: 16/ENG01/023

Using the Leibnitz theorem, given that

i) $y = x^3 e^{4x}$ determine y^5

$u = e^{4x}$ $v = x^3$

Solution.

$u^5 = 4^5 e^{4x}$ $v' = 3x^2$

$u^4 = 4^{5-1} e^{4x} = 4^4 e^{4x}$ $v'' = 6x$

$u^3 = 4^{5-2} e^{4x} = 4^3 e^{4x}$ $v''' = 6$

$u^2 = 4^{5-3} e^{4x} = 4^2 e^{4x}$ $v'''' = 0$

$u^1 = 4^{5-4} e^{4x} = 4^1 e^{4x}$

$u^0 = 4^{5-5} e^{4x} = 4^0 e^{4x}$

$= e^{4x}$

$$y^5 = \left\{4^5 e^{4x} \cdot x^3\right\} + \left\{5 \cdot 4^4 e^{4x} \cdot 3x^2\right\} + \left\{\frac{5(5-1)}{2!} \cdot 4^3 e^{4x} \cdot 6x\right\}$$

$$+ \left\{\frac{5(5-1)(5-2)}{3!} \cdot 4^2 e^{4x} \cdot 6\right\}$$

$$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}$$

$$y^5 = e^{4x}\left\{1024x^3 + 3840x^2 + 3840x + 960\right\}.$$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

Show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$.

Solution

$x^2y^2 + xy^1 + y^0 = 0$.

$\downarrow$ w_1 $\downarrow$ w_2 $\downarrow$ w_3

$w_1 \Rightarrow$ $u = y^2$ $v = x^2$

$u^n = y^{n+2}$ $v' = 2x$ $w_1^n = y^{(n+2)} \cdot x^2 + n \cdot y^{(n+1)} \cdot 2x$

$u^{n-1} = y^{n+1}$ $v'' = 2$. $+ \frac{n(n-1)}{\cancel{2!}} \cdot y^n \cdot \cancel{2}$

$u^{n-2} = y^n$ $v''' = 0$

$$w_1^n = x^2 y^{n+2} + 2xn\, y^{(n+1)} + n(n-1)y^n$$

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$W_2 \Rightarrow \quad u = y' \quad v = x \qquad W_2^n = y^{n+1} \cdot x + n \cdot y^n \cdot 1$

$u^n = y^{n+1} \quad v' = 1$

$u^{n-1} = y^n \quad v'' = 0. \qquad W_2^n = xy^{(n+1)} + ny^n$

$W_3 \Rightarrow \quad u = y^0 \quad v = 1 \qquad W_3^n = y^n \cdot 1$

$u^n = y^n \quad v' = 0. \qquad W_3^n = y^n$

$W_1 + W_2 + W_3 = 0$

$x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n = 0$

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0.$

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$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = \underbrace{(2x+1)(2x+1)e^{x^2+x}}_{y'} + \underbrace{2e^{x^2+x}}_{y}$

$y'' = (2x+1)y' + 2y$

$\underbrace{y''}_{w_1} - \underbrace{(2x+1)y'}_{w_2} - \underbrace{2y}_{w_3} = 0$

$w_1 \Rightarrow \quad u = y^2 \quad v = 1$

$u^n = y^{n+2} \quad v' = 0$

$w_1^n = y^{n+2}$

$w_2 \Rightarrow \quad u = y' \quad v = -(2x+1)$

$u^n = y^{n+1} \quad v' = -2$

$u^{n-1} = y^n \quad v'' = 0$

$w_2^n = -(2x+1)y^{n+1} + n(-2)y^n$

$w_3 \Rightarrow \quad u = y^0 \quad v = -2$

$u^n = y^n \quad v' = 0$

$w_3^n = -2y^n$

$w_1 + w_2 + w_3 = 0$

$y^{n+2} + (-(2x+1))y^{n+1} - 2ny^n - 2y^n = 0$

$y^{n+2} = +(2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2y^n(n+1)$