AGBEDE GOODNESS YERIMMINI'
15/ENG03/003
CIVIL ENGINEERING
ENG 381 (ENGINEERING MATHEMATICS)

1. $y = e^{x^2+x}$ ——— (i)

$v = 1$, $v' = 0$

$u = e^{x^2+x}$ $u^n = (2x+1)^n e^{x^2+x}$

$y' = u^n \cdot v + 0$

$y' = (2x+1)^n e^{x^2+x}$ ——— (ii)

from y'

$v = 2x+1$, $v' = 2$ $v'' = 0$

$u^n = (2x+1)^n e^{x^2+x}$

$y'' = u^n \cdot v + nu^{(n-1)} v' + 0$

$y'' = (2x+1)^n e^{x^2+x} \cdot (2x+1) + n(2x+1)^{1-1} e^{x^2+x}$

$y'' = (2x+1)^n e^{x^2+x} \cdot (2x+1) + n(2x+1)^0 e^{x^2+x}$

Sub equation (i) and (ii)

$y'' = y'(2x+1) + 2y$

ii Prove that $y^{(n+2)} = (2x+1)y^{n+1} + 2(n+1)y^n$.

where $w = y''$

from above $y'' = y'(2x+1) + 2y$

$u = 1$ $v = 1$

$v = y''$ $u^n = y^{(n+2)}$

$w^n = u^n \cdot v + 0$

$w^n = y^{n+2}$

where $w^n = y'(2x+1)$

$v = 2x+1$ $v' = 2$ $v'' = 0$

$u = y'$ $u^n = y^{(n+1)}$

$w^n = u^n \cdot v + nu^{(n-1)} v' + 0$

$w^n = y^{(n+1)} \cdot 2x+1 + ny^{n+1-1} \cdot 2$

$w^n = (2x+1) y^{n+1} + 2ny^n$.

where $w = 2y$

$v = 2 \quad v' = 0$

$u = y \quad u^n = y^n$

$w^n = u^n v + 0$

$w^n = 2 \cdot y^n$

$y^n = y^{(n+2)} - y^{(n+1)}(2x+1) + n2y^n - 2y^n = 0$

$\therefore y^{(n+2)} = y^{(n+1)}(2x+1) + n2y^n + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^n(n+1)$

2. Using the Leibnitz theorem given that

(i) $y = x^3 e^{4x}$ determine $y^{(5)}$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ show that

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Solution

$y = x^3 e^{4x}$

$v = x^3 \quad u = e^{4x}$

$v' = 3x^2 \quad u^{(n)} = 4^n e^{4x}$

$v'' = 6x$

$v''' = 6$

$v^{iv} = 0$

Using Leibnitz theorem

$$y^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}v''}{2!} + \frac{n(n-1)(n-2)u^{(n-3)}v'''}{3!}$$

$+0$

$$y^n = 4^n e^{4x} \cdot x^3 + n4^{n-1}e^{4x} \cdot 3x^2 + \frac{n(n-1)4^{n-2}e^{4x} \cdot \cancel{6}x}{\cancel{2} \times 1}$$

$$+ \frac{n(n-1)(n-2)4^{(n-3)}e^{4x} \cdot \cancel{6}x}{3 \times \cancel{2} \times 1}$$

$y^{(n)} = x^3 4^n e^{4x} + 3x^2 n 4^{(n-1)} e^{4x} + n(n-1)4^{(n-2)}e^{4x} \cdot 3x$

$+ n(n-1)(n-2)4^{(n-3)}e^{4x}$

$y^{(5)} = x^3 \cdot 4^5 e^{4x} + 3x^2 \cdot 5 \cdot 4^{(5-1)} e^{4x} + 5(5-1)4^{(5-2)}e^{4x} \cdot 3x$

$+ 5(5-1)(5-2)4^{(5-3)}e^{4x}$

$y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x}$

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

$x^2y'' + xy' + y = 0.$

1st product => x^2y''.

$v = x^2 \quad v' = 2x \quad v'' = 2 \quad v''' = 0$

$u = y'' \quad u^{(n)} = y^{n+2}.$

$y^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}}{2!}v'' + \frac{n(n-1)(n-2)}{3!}u^{n-3}v^3$

$y = y^{n+2}\cdot x^2 + nu^{(n-2)}\cdot 2x + \frac{n(n-1)}{2\times 1}u^{(n-2)}\cdot 2 + \frac{n(n-1)(n-2)}{3\times 2\times 1}u^{(n-3)}\cdot 0$

$= x^2y^{n+2} + 2xnu^{(n-1)} + n(n-1)u^{(n-2)}$

$= x^2y^{(n+2)} + 2nxy^{n+2-1} + n(n-1)y^{n+2-2}.$

$= x^2y^{(n+2)} + 2nxy^{n+1} + n(n-1)y^n$

2nd product =>.

xy'

$v = x \quad v' = 1 \quad v'' = 0$

$u = y' \quad u^{(n)} = y^{n+1}$

$= u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v''$

$= u^{(n)}v + nu^{(n-1)}$

$= y^{n+1}\cdot x + ny^{n+1-1}$

$= xy^{n+1} + ny^n$

y =>.

$v = 1 \quad v' = 0$

$u = y \quad u^{(n)} = y^n$

$= u^{(n)}v + nu^{(n-1)}v'$

$= u^{(n)}v = y^n$

$= x^2y^{(n+2)} + 2nxy^{n+1} + n(n-1)y^n + xy^{n+1} + ny^n + y^n$

$= x^2y^{(n+2)} + (2n+1)xy^{n+1} + y^n(n^2+1) = 0.$