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ENG381

1) If $y = e^{x^2} + x$
$u = x^2 + x$
$\frac{du}{dx} = 2x + 1$

$y = e^u$
$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$= e^u \times 2x+1$
$2x+1e^u$ $\quad u = x^2 + x$
$\frac{dy}{dx} = 2x+1e^{x^2+x}$
$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$
$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$
$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2 + 4x + 1 + e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2}$, $y' = \frac{dy}{dx}$ $\quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$
$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$
$y' = (2x+1) = (2x+1)(2x+1)e^{x^2+x}$
$= 4x^2 + 4x + 1e^{x^2+x}$

$2y = 2e^{x^2+x}$
$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$
$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

w_1

$u = y^n$ $\qquad v = 1$

$u^n = y^{n+1}$ $\qquad v = 0$

$= y^{n+1} \cdot 1 + 0$

w_2

$u = y^n$ $\qquad v = 2x+1$

$u^n = y^{n+1}$ $\qquad v' = 2$

$u^{n-1} = y^n$ $\qquad v = 0$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

w_3

$u = xy$ $\qquad v = 1$

$u^n = y^n$ $\qquad v' = 0$

$= 2[y^n - 1) + 0]$

$= 2y^n$

$w_1 = w_2 + w_2$

$y^{n+1} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2a) Using the Leibnitz theorem gives that $y = x^3 e^{4x}$ determine $y^{(5)}$

Solutions

$= 10240 e^{4x} x^3 + 12800 e^{4x} \cdot 3x^2 + 6400 e^{4x} \cdot 6x + 800 e^{4x} \cdot 6$

$= 10240 e^{4x} x^3 + 38400 e^{4x} x^2 + 38400 e^{4x} + 4800 e^{4x}$

(4) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

$\downarrow \quad \downarrow \quad \downarrow$

$w_1 \quad w_2 \quad w_3$

$w_1 + w_2 + w_3 = 0$

for w_1

$u = y''$ $\quad v = x^2$

$u^n = y^{n+2}$ $\quad v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$= y^{(n+2)} (x^2) + n (y^{n+1}) 2x + \frac{n(n-1) y^n \cdot 2}{2!} = 0$

$= x^2 y^{(n+2)} + 2nx (y^{n+1}) + n(n-1) y^n$

for w_2

$w = y'$ $\quad u = x$