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15/ENG06/049
Mechanical Eng.

(1) $y = e^{x^2+x}$; Show that : $y'' = y'(2x+1) + 2y$

Soln

$y'' = y'(2x+1) + 2y$ ——— (*)

$y'' = \frac{d^2y}{dx^2}$ $y' = \frac{dy}{dx}$

$\therefore dy/dx = (2x+1)e^{x^2+x}$

$d^2y/dx^2 = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1+2)e^{x^2+x}$

$= (4x^2+4x+3)e^{x^2+2x}$

Substituting RHS & LHS above eqn *

$(4x^2+4x+3)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1+3)e^{x^2+x}$

$= (4x^2+4x+4)e^{x^2+x}$

$(4x^2+4x+3)e^{x^2+x} = (4x^2+4x+3)e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$ //

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II $y'' = y'(2x+1) + 2y$

$y'' = y(2x+1) - 2y = 0$

Let $W = y'$

$v = 1 \quad v' = 0 \quad u = y'' , \; u^n = y^{n+1}$

$W^n = u^n v + u^{n+1} v'$

$= y(n+2) + 0$

Let $W = -y'(2x+1)$

$v = 2x+1 \quad v' = 2 \quad v'' = 0$

$u = -y' \quad u^n = -y^{n+1}$

$$W = u^n v + u^{n-1} v' + \frac{n(n-1) u^{n-2} v''}{2!}$$

$= -y^{n+1}(2x+1) + n(-y^{n+1})(2) + 0$

$= -y^{n+1}(2x+1) + 2n(-y^n)$

Let $W = -2y$

$v = -2 \quad v' = 0$

$u = y \quad u^n = y^n$

$W^n = u^n v + u^{n-1} v'$

$= y^n - 2 + 0$

$= -2y^n$

$\therefore y^n = y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^2$

2. $y = x^3 e^{4x}$ find y^3

$V = x^3$, $V' = 3x^2$, $V'' = 6x$, $V''' = 6$, $V^{iv} = 0$

$u = e^{4x}$, $u' = 4e^{4x}$ $u^2 = 16e^{4x}$, $u^3 = 64e^{4x}$, $u^4 = 256e$

$u^5 = 1024e^{4x}$

$y^n = u^n v + V^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}$

$u^{n-3}v^3 + \frac{n(n-1)(n-2)(n-3)}{4!}u^{n-4}v^4$

$y^5 = u^5 v + 5u^4 v' + \frac{5(4)}{2\times1}u^3 v'' + \frac{5(4)(3)}{3\times2\times1}u^2 v^3 + 0$

$y^5 = 1024e^{4x}(x^3) + 15x^2(256e^{4x}) + 60x(64e^{4x})$

$+ 60(6e^{4x})$

~~$y^5 = 1024e^{4x}(x^3) + 15x^2(256e^{4x}) + 60x$~~

$y^5 = x^3 1024e^{4x} + x^2 3840e^{4x} + x\, 3840e^{4x} + 960e$

$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$

2(i) $x^2 \, dy/dx + x \, dy/dx + y = 0$

$\Rightarrow x^2 y'' + xy' + y = 0$

let $W = x^2 y''$

$V = x^2$, $V = 2x$ $V'' = 2$, $V''' = 0$

$U = y''$. $u^n = y^{n+2}$

$W^n = u^n v + n u^{n-1} v - \frac{n(n-1)}{2!}u^{n-2}v'' + \frac{n(n-1)(n-2)}{3!}u^n$

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$= y^{n+2}(x^2) + n(y^{n+2-1})(2x) + \frac{n(n-1)(y^{n+2-2})}{2!}$

$= x^2 y^{n+2} + 2xn(y^{n+1}) + n(n-1)(y^n)$

let $W = xy$

$V = x, \quad V' = 1 \quad V'' = 0$

$u = y' \qquad u^n = y^{n+1}$

$W^n = y^{n+1}(x) + n(y^{n+1-1})(1) + 0$

$= x y^{n+1} + n y^n$

Let $W = y$

$V = 1, \quad V' = 0$

$u = y \quad u^n = y^n$

$W^n = y^n$

$y^n = x^2 y^{(n+2)} + 2xn(y^{n+1}) + n(n-1)(y$

$x^2 y(n+2) + (2n+1)xy(n+1) + (n^2 - n + n + 1)$

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$