Assignment Macdonald Perez Computer 15/Eng02/035

1) $y = e^{x^2+x}$: show that $y'' = y'(2x+1) + 2y$

$y'' = y'(2x+1) + 2y$ ----

$y'' = d^2y/dx^2 \qquad y' = \frac{dy}{dx}$

$\therefore dy/dx = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = (2x+1)(2x+1)e^{x^2+x} + 2 \cdot e^{x^2+x}$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1) e^{x^2+x}$

$= (4x^2+4x+3) e^{x^2+2x}$

Sub RHS & LHS

$(4x^2+4x+3)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$= (2x^x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1+3) e^{x^2+x}$

$= (4x^2+4x+4) e^{x^2+x}$

$(4x^2+4x+3)e^{x^2+x} = (4x^2+4x+3)e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$ //

ii) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Let $w = y''$

$v = 1 \qquad v' = 0$

$u = y'' \qquad u^n = y^{n+2}$

$w^n = u^n v + n u^{n-1} v'$

$= y^{(n+2)} + 0$

Let $w = -y'(2x+1)$

$v = 2x+1 \qquad v' = 2 \qquad v'' = 0$

$u = -y'$

$w^n = u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v''$ $\quad u^n = -y^{n+1}$

$= -y^{n+1}(2x+1) + n(-y^{n+1-1})(2) + 0$

$= -y^{n+1}(2x+1) + 2n(-y^n)$

let $w = -2y$

$v = -2 \quad v' = 0$

$u = y \quad u^n = y^n$

$w^n = u^n v + n u^{n-1} v'$

$= y^n \cdot -2 + 0$

$= -2y^n$

$\therefore y^n = y^{n+2} = -y^{n+1}(2x+1) + 2n(-y^n) - 2y^n$

$y^{n+2} = -y^{n+1}(2x+1) + 2n(-y^n) - 2y^n = 0$

$y^{n+2} + y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^n(n-1)$ //

2) $y = x^3 e^{4x}$ find y^5

$v = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$, $v'''' = 0$

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u''' = 64e^{4x}$, $u'''' = 256e^{4x}$, $u^5 = 1024e^{4x}$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v''' +$

$\frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^4$

$y^5 = 1024 e^{4x}(x^3) + 15x^2(256e^{4x}) + 60x(64e^{4x}) + 60(16e^{4x})$

$y^5 = x^3 1024 e^{4x} + x^2 3840 e^{4x} + x 3840 e^{4x} + 960 e^{4x}$

$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$ //

3) $x^2 \, d^2y/dx^2 + x \, dy/dx + y = 0$

$x^2 y'' + xy' + y = 0$

let $w = x^2 y''$

$v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$u = y''$, $u^n = y^{n+2}$

$w^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'''$

$= y^{n+2}(x^2) + n(y^{n+1})(2x) + \frac{n(n-1)}{2!}(y^{n+2-2})(2)$

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$-x^2y^{n+2} + 2xn(y^{n+1}) + n(n-1)(y^n)$

Let $w = xy$

$v = x$, $v' = 1$, $v'' = 0$

$u = y$, $u^n = y^{n+1}$

$w^n = y^{n+1}(x) + n(y^{n+1-1})(1) + 0$

$= xy^{n+1} + ny^n$

Let $w = y$

$v = 1$ $\quad v' = 0$

$u = y$ $\quad u^n = y^n$

$w^n = y^n$

$y^n = x^2y^{(n+2)} + 2xn(y^{n+1}) + n(n-1)(y^n) + xy^{n+1} + ny^n + y^n$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-n+n+1)y^n = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$