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15/ENG06/013
MECHANICAL ENGINEERING

1) If $y = e^{x^2+x}$

Show that $y = y(2x+1)+2y$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2y + (2x+1)y'$

Hence $y'' = y'(2x+1)+2y$

$y^{(n+2)} = y^{(n+1)}.(2x+1) + (n+1)y^n.2$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) $y = x^3e^{4x}$

determine y^5

$y^n = u^nv + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2$

$+ \frac{n(n-1)(n-2)}{3!}v^3 + \ldots$

Let $v = x^3$ and $u = e^{4x}$

$y^5 = (4)^5e^{4x}.x^3 + 5(4)^4e^{4x}.3x^2 +$
$10(4)^3e^{4x}.6x + 10(4)^2e^{4x}.6 + 0$

$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} +$
$3840xe^{4x} + 960e^{4x}$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

3) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that ~~x^3f~~

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$x^2y'' + xy' + y = 0$

$x^2y^{(2)} + xy^{(1)} + y = 0$

Using Leibnitz theorem

$y^n = y^{(n+2)}.x^2 + n.2xy^{(n+1)} +$
$\frac{2n(n-1)}{2!}y^n + y^{(n+1)}.x + ny^n.1 + y^n$

$y^n = x^2y^{(n+2)} + 2xy^{(n+1)}n + n(n-1)y^n$
$+ xy^{(n+1)} + ny^{(n)} + y^{(n)}$

$y^n = y^{(n+2)}(x^2) + y^{(n+1)}(2xn + x)$
$+ y^{(n)}(n(n-1) + n - 1)$

$y^n = x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2-1)y^n$

$\therefore x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-1)y^n = 0$