

19 EN 057010

GATEWAY INSTITUTIONS

Petrolium Engineering

$$① y = e^{2x+2}$$

$$\frac{dy}{dx} = (2x+1)e^{2x+2}$$

$$\frac{d^2y}{dx^2} = 2e^{2x+2} + (2x+1)(2)e^{2x+2}$$

$$= 2e^{2x+2}((2x+1)(2)+2)$$

$$y' = (2x+1)e^{2x+2} + 2y$$

$$(2x+1)e^{2x+2}((2x+1)+2)$$

$$= e^{2x+2}((2x+1)(2x+3)+2)$$

$$y' = y' (2x+1) + 2y$$

$$w_1$$
$$u = y^2$$
$$v^n = y^{(n+1)}$$

$$w_2$$
$$u = y' \quad v = 2x+1$$

$$u^n = y^{n+1} \quad v' = 2$$

$$u^{n-1} = y^n \quad v^n = 0$$

⑥ $y = z^3 e^{4x}$

$u = e^{4x}$

$u^n = 4^n e^{4nx}$

$u^{n+1} = 4^{n+1} e^{4(n+1)x}$

$u^{n+2} = 4^{n+2} e^{4(n+2)x}$

$u^{n+3} = 4^{n+3} e^{4(n+3)x}$

$v = z^3$

$v' = 3z^2$

$v'' = 6z$

$v''' = 6$

$v^{(4)} = 0$

$= 4^n e^{4nx} z^3 + n 4^{n+1} e^{4(n+1)x} \cdot 3z^2 + \frac{n(n+1)}{2!} 4^{n+2} e^{4(n+2)x} z$

$\frac{(z + n(n+1)(n-1)) 4^{n+2} e^{4nx}}{2!} z$

$n = 1$

$= 4^1 e^{4x} z^3 + 5 \cdot 4^2 e^{8x} \cdot 3z^2 + 10 \cdot 4^3 e^{12x} z + 10 \cdot 4^4 e^{16x} z^3 + 6$

⑦ $z^2 \frac{d^2 y}{dz^2} + z \frac{dy}{dz} + y = 0$

Show that $z^2 y^{(n+2)} + (2n+1) z y^{(n+1)} + (n^2+1) y^{(n)} = 0$

$z^2 y^{(n)} + 2(z y)^{(n)} + y = 0$

$w_1 \quad w_2 \quad w_3$

w_1

$y = y^{(n)}$

$y^{(n+1)} = y^{(n+1)}$

$y^{(n+2)} = y^{(n+2)}$

$y^{(n+3)} = y^{(n+3)}$

$v = z^2$

$v' = 2z$

$v'' = 2$

$v^{(3)} = 0$

$$\begin{aligned}
 u &= y^2 & r &= 2 \\
 u' &= 2y & r' &= 1 \\
 u'' &= 2 & r'' &= 0
 \end{aligned}$$

$$\begin{aligned}
 u &= y \\
 u' &= y^n
 \end{aligned}$$

$$\begin{aligned}
 &w_1 \text{ var } w_2 \text{ var } w_3 \\
 &= y^{nr} \cdot 2^n + n y^{nr-1} \cdot 2 + \frac{n(n-1)}{2!} y^{nr-2} \cdot 2 + y^{nr-2} \cdot 2 + n y^{nr-1} + y^{nr} = 0
 \end{aligned}$$

$$\begin{aligned}
 &= 2^n y^{nr} + 2n y^{nr-1} + n(n-1) y^{nr-2} + 2 y^{nr-2} + n y^{nr-1} + y^{nr} = 0 \\
 &= 2^n y^{nr} + 2n y^{nr-1} + 2 y^{nr-2} + n(n-1) y^{nr-2} + n y^{nr-1} + y^{nr} \\
 &= 2^n y^{nr} + 2 y^{nr-2} (n-1) + y^{nr} (n-1) (n+1) \\
 &= 2^n y^{nr} + 2 y^{nr-2} (n+1) + y^{nr} (n^2 - n + n - 1) \\
 &= 2^n y^{nr} + 2 y^{nr-2} (n+1) + y^{nr} (n^2 + 1) = 0
 \end{aligned}$$