SMOOTH SIMON

15/ENG03/030

CIVIL ENG.

ENG 381.

① If $y = e^{x^2+x}$

$y = x^2 + x$

$\frac{dy}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{dx} = \frac{dy}{dx} \times \frac{du}{dx}$

$= e^u \times 2x + 1$

$2x + 1e^u$

$\frac{dy}{dx} = 2x + 1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}.$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} \quad y' = \frac{dy}{dx} \quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + 1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}.$

$\underbrace{y''}_{w_1} \quad \underbrace{y'(2x+1)}_{w_2} + \underbrace{2y}_{w_3}$

w_1

$u = y''$ $\qquad$ $u^{d}; v = 0$

$u^{n} = y^{n+2}$

$= y^{n+2}, + 0$

w_2

$u = y'$ $\qquad$ $v = 2x + 1$

$u^{n} = y^{n+1}$ $\qquad$ $v' = 2$

$u^{n-1} = y^{n}$ $\qquad$ $v = 0$

$= y^{n+1}(2x + 1) + n(y^{n}) = 10$

$= y^{n+1}(2x + 1) + 2n(y^{n})$

w_3

$y = 2y$ $\qquad$ $v = 1$

$u^{n} = y^{n}$ $\qquad$ $v' = 0$

$= 2[(y^{n} \cdot 1) + 0]$

$= 2y^{n}.$

$w_1 = w_2 + w_3$

$y^{n+2} = y^{n+1}(2x + 1) + 2n(y^{n}) + 2y^{n}$

$= y^{n+1}(2x + 1) + 2(n + 1)y^{n}.$

(2a) Using the leibinitz theorem given that $y = x^{3}e^{4x}$ determine y^{5}.

Solution

$y = e^{4x}$ $\quad$ $v = x^{3}$

$y^{(5)} = y^{(5)}v + 5u^{4}v' + 10u^{3}v'' + 10^{2}u^{2}v^{3} + 5u'v^{4} + uv^{5}$

$= u^{5}e^{4x}x^{3} + 5(4^{4}e^{4x}3x^{2}) + 10(4^{3}e^{4x}6x) + 5(4^{2}e^{4x} \cdot 6) + 0$

$= 1024e^{4x}x^{3} + 1280e^{4x} \cdot 3x^{2} + 640e^{4x} \cdot 6x + 80e^{4x} \cdot 6$

$= 1024e^{4x} + 3840e^{4}x^{2} + 3840e^{4x} + 480e^{4x}.$

$x^{2}\frac{dy^{2}}{dx^{2}} + x\frac{dy}{dx} + y = 0$

$$x^2y'' + xy' + y = 0$$

$w_1 \quad w_2 \quad w_3$

$$w_1 + w_2 + w_3 = 0$$

for w_1

$u = y''$ $\quad v = x^2$

$u^n = y^{n+2}$ $\quad v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$u^{n-2} = y^n$ $\quad v''' = 0$

$$= y^{(n+2)}(x^2) + n(y^{n+1})\,2x + \frac{n(n-1)y^2}{2!} + 0$$

$$= x^2y(n+2) + 2nx(y^{n+1}) + n(n-1)y^n$$

for w_2

$u = y'$ $\quad v = x$

$u^n = y^{n+1}$ $\quad v' = 1$

$u^{n-1} = y^n$ $\quad v'' = 0$

$$= y^{n+1}x + nyn + 0$$

for w_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$$= y^n \cdot 1$$

$$w_1 + w_2 + w_3 = 0$$

$$x^2y^{n+2} + 2nxy^{n+1} + (n^2-n)y^n + xy^{n+1} + ny^n + y^n$$

$$x^2y^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2y^n \cancel{- ny^n} + \cancel{ny^n} + y^n$$

$$x^2y^{n+2} + 2n + 1(xy^{n+1}) + (n^2+1)y^n.$$

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