Name; Ejemeka Chiderauwaoma Cynthia
matric no; 19lENG071015
Department; Petroleum Engineering

1 $y = e^{x^2+x}$ -- ⓐ

$y^n = a^n e^{ax}$

$y' = (2x+1)e^{x^2+x}$ -- ⓑ

$u = 2x+1 \qquad v = e^{x^2+x}$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} (2x+1)e^{x^2+x}$

using product rule

$y'' = \frac{u\,dv}{dx} + \frac{v\,du}{dx}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

from equ a and b

$y'' = y'(2x+1) + 2y$ ....

$y^{(2)} = y^{(1)}(2x+1) + 2y$

$w_1 = y^{(2)}$ $\qquad w_2 = y^{(1)}(2x+1)$ $\qquad w_3 = 2y$

$u = y^{(2)} \quad v = 1$ $\qquad u = y^{(1)} \quad v = 2x+1$ $\qquad u = y \quad v =$

$u^n = y^{(2+n)}$ $\qquad u^n = y^{(1+n)} \quad v' = 2$ $\qquad u^n = y^n$

$u^{n-1} = y^{(n)}$

$w_1^{(n)} = w_2^{(n)} + w_3^{(n)}$

$y^n = u^{(n)}v + nu^{(n-1)}v'$

$y^{(2+n)} = y^{(1+n)} \cdot (2x+1) + n(y^{(n)}) \cdot 2 + y^{(n)} \cdot 2$

$y^{(2+n)} = (2x+1)y^{(1+n)} + 2(n+1)y^{(n)}$

2a) $y = x^3 e^{4x}$ find $y^{(8)}$

$u = e^{4x} \qquad v = x^3$

$u^n = 4^n e^{4x} \qquad v' = 3x^2$

$u^{n-1} = 4^{n-1}e^{4x} \qquad v^2 = 6x$

$u^{n-2} = 4^{n-2}e^{4x} \qquad v^3 = 6$

$u^{n-3} = 4^{n-3} e^{4x}$

$y^{(n)} = u^{(n)}v + n u^{(n-1)} v' + \frac{n(n-1) u^{(n-2)}}{2!} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)}$

2a) $y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x}) \cdot 3x^2 + \frac{5(4)}{2!}(4^3 e^{4x}) \cdot 3 \cancel{x} + \ldots$

$y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840xe^{4x} + 3840xe^{4x} + 960e^{4x}$

$y^{(5)} = 64e^{4x} [16x^3 + 60x^2 + 60x + 15)$.

2b) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$w_1 = x^2 y^{(2)}$

$u = y^{(2)}$ $v = x^2$

$u^{(n)} = y^{(2+n)}$ $v' = 2x$

$u^{(n-1)} = y^{(1+n)}$ $v^2 = 2$

$u^{(n-2)} = y^{(n)}$

$w_2 = xy^{(1)}$

$u = y^{(1)}$ $v = x$

$u^{(n)} = y^{(1+n)}$ $v' = 1$

$u^{(n-1)} = y^{(n)}$

$w_3 = y$

$u = y$ $v = 1$

$u^n = y^{(n)}$

using lebnitz theorem

$y^n = u^{(n)}v + nu^{(n-1)}v^{(1)} + \frac{n(n-1) u^{(n-2)} v^{(2)}}{2!}$

$w_1(n) = x^2 y^{(n+2)} + 2xny^{(1+n)} + n(n-1)y^{(n)}$

$w_2(n) = y^{(1+n)} \cdot x + ny^{(n)} \cdot 1$

$$w_2^{(n)} = xy^{(1+n)} + ny^{(n)}$$

$$w_3^{(n)} = y^{(n)} \cdot 1 = y^{(n)}$$

adding together;

$$x^2 y^{(n+2)} + 2xny^{(1+n)} + n(n-1)y^{(n)} + xy^{(1+n)} + ny^{(n)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + xy^{(1+n)}[2n+1] + y^{(n)}[n(n-1) + n+1] = 0$$

$$x^2 y(n+2) + xy^{(1+n)}[2n+1] + y^{(n)}[n^2 - n + n + 1] = 0$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(1+n)} + (n^2+1)y^{(n)}$$