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15/ENG06/045

1. $y = e^{x^2+x}$

$u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times 2x + 1$

but $u = x^2 + x$

$\frac{dy}{dx} = 2x + 1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2}$ $y' = \frac{dy}{dx}$ $y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + 1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1)+2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$$y'' = y'(2x+1) + 2y$$

$$\underset{\omega_1}{\downarrow} \qquad \underset{\omega_2}{\vee} \qquad \underset{\omega_3}{\downarrow}$$

$\omega_1$

$u = y''$ $\qquad v = 1$

$u^n = y^{n+2}$ $\qquad v = 0$

$= y^{n+2} \cdot 1 + 0$

$\omega_2$

$u = y'$ $\qquad v = 2x+1$

$u^n = y^{n+2}$ $\qquad v' = 2$

$\cancel{y^{n+2} \cdot 1 = 0}$ $\qquad v'' = 0$

$u^{n-1} = y^n$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

$\omega_1 = \omega_2 + \omega_3$

$= y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2a Using of Leibnitz theorem given that $y = x^3 e^{4x}$ determine $y^{(5)}$

Soln

$u = e^{4x}$ $\quad\quad v = x^3$

$y^{(5)} = (uv)^{(5)}$

$y^{(5)} = u^{(5)}v + 5u^4v' + 10u^3v'' + 10u^2v''' + 5u'v^4 + uv^5$

$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} 6x) + 5(4^2 e^{4x} 6) + 0$

$= 1024 e^{4x} x^3 + 3840 e^{4} x^2 + 3840 e^{4x} x + 480 e^{4x}$

ii; $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$$\underbrace{x^2 y''}_{\omega_1} + \underbrace{xy'}_{\omega_2} + \underbrace{y}_{\omega_3} = 0$$

$\omega_1 + \omega_2 + \omega_3 = 0$

For $\omega_1$

$u = y''$ $\quad\quad v = x^2$

$u^n = y^{n+2}$ $\quad\quad v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad\quad v'' = 2$

$u^{n-2} = y^n$ $\quad\quad v''' = 0$

$= y^{(n+2)}(x^2) + n(y^{n+1})2x + n(n-1)y^n \cdot x + 0$

$x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$

For $\omega_2$

$u = y'$ $\quad\quad v = x$

$u^n = y^{n+1}$ $\quad\quad v' = 1$

$u^{n-1} = y^n$ $\quad\quad v'' = 0$

$y^{n+1} \cdot x + ny^n + 0$

For $w_3$

$y = y$ $\qquad$ $y = 1$

$y^n = y^n$ $\qquad$ $y' = 0$

$y^{n+1}$

$w_1 + w_2 + w_3 = 0$

$x^2 y^{n+2} + 2nx y^{n+1} + (n^2 - n) y^n + xy^{n+1} + ny^n + y^n$

$xy^{n+2} + 2nx y^{n+1} + xy^{n+1} + n^2 y^n - ny^n + ny^n + y^n$

$x^2 y^{n+2} + (2n+1)(xy^{n+1}) + (n+1) y^n$