

NAME OPARA NDAMENIUM
 MATRIC 15/ENIG06/057
 DEPT MECHANICAL

i)

$$y = e^{x^2+x}$$

soln

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$$

$$y'' = 2y + (2x+1)y'$$

$$y'' = y'(2x+1) + 2y$$

$$y^{(n+2)} = y^{(n+1)} \cdot (2x+1) + (n+1)y^n \cdot 2$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

ii)

$$y = x^3 e^{4x}$$

$$y^{(n)} = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} v^3 + \dots +$$

$$\text{let } x^3 = v \text{ and } u = e^{4x}$$

$$y^{(5)} = (4)^5 e^{4x} \cdot x^3 + 5(4)^4 e^{4x} \cdot 3x^2 + 10(4)^3 e^{4x} \cdot 6x + 10(4)^2 e^{4x} \cdot 6 + 0$$

$$y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x} + 0$$

$$y^{(5)} = e^{4x} (1024x^3 + 3840x^2 + 3840x + 960)$$

ii)

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$x^2 y'' + x y' + y = 0$$

$$x^2 y^{(2)} + x y^{(1)} + y^{(0)} = 0$$

Using Leibnitz theorem

$$y^n = y^{(n+2)} \cdot x^2 + n \cdot 2xy^{(n+1)} + \frac{2n(n-1)}{2!} y^n + y^{(n+1)} \cdot x + n \cdot y^{(n)} \cdot 1 + y^n$$

$$y^n = x^2 y^{(n+2)} + 2xy^{(n+1)}$$

$$y^n = x^2 y^{(n+2)} + 2xy^{(n+1)} \cdot n + n(n-1)y^n + x y^{(n+1)} + n y^{(n)} + y^{(n)}$$

$$y^n = y^{(n+2)}(x^2) + y^{(n+1)}(2xn+x) + y^{(n)}(n(n-1)+n+1)$$

$$y^n = x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2 - n + n + 1)y^{(n)}$$

$$y^n = x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2 - 1)y^{(n)}$$

equale to 0

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2 - 1)y^{(n)} = 0$$