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15/ENG06/056
Mechanical Engineering

1) If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$

soln

$y = e^{x^2+x}$

$\frac{dy}{dx} = y'$

let $y = e^u$, $dy/dx = y' = dy/du \cdot du/dx$

$\frac{dy}{dx} = y' = e^u \times 2x+1$

$y' = (2x+1)e^u$

$y' = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = y''$

$y' = (2x+1)e^{x^2+x}$

Using Product Rule

let $u = 2x+1$ $\quad v = e^{x^2+x}$

$\frac{du}{dx} = 2$ $\quad \frac{dv}{dx} = 2x+1e^{x^2+x}$

$\therefore \frac{d^2y}{dx^2} = y'' = v\frac{du}{dx} + u\frac{dv}{dx}$

$= 2(e^{x^2+x}) + 2x+1(2x+1e^{x^2+x})$

Since $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2(y) + (2x+1)(y')$

$y'' = 2y + (2x+1)y'$

$y'' = y'(2x+1) + 2y$

b) Prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

$y'' = y'(2x+1) + 2y$

$y' = y'(2x+1) - 2y = 0$

Using leibnitz Theorem

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i) y''

$u = y''$ $\qquad v = 1$

$u^n = y^{n+1}$ $\qquad v' = 0$

$= u^n v + n u^{n-1} v'$

$= y^{n+2}(1) + n(y^{(n+1)})(0)$

$= y^{n+2}$

ii) $-y'(2x+1)$

$u = y'$ $\qquad v = -(2x+1)$

$u^n = y^{n+1}$ $\qquad v' = -2$

$\qquad v^n = 0$

$= u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v^n$

$= -y^{n+1}(+2x+1) + n(y^n)(-2)$

$= -(2x+1)y^{n+1} - 2ny^n$

iii) $-2y$

$u = y$ $\qquad v = -2$

$u^n = y^n$ $\qquad v' = 0$

$= u^n v + n u^{n-1} v'$

$= -2y^n$

$\therefore y^{n+2} = -(2x+1)y^{n+1} - 2ny^n - 2y^n = 0$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + 2y^n(n+1)$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) Using the leibnitz theorem given that

$y = x^3 e^{4x}$

determine $y^{(5)}$

$v = x^3$ $\qquad u = e^{4x}$

$v' = 3x^2$ $\qquad u' = 4e^{4x}$

$v'' = 6x$ $\qquad u'' = 16e^{4x}$

$v^3 = 6$ $\qquad u''' = 64e^{4x}$

$v^4 = 0$ $\qquad u^4 = 256e^{4x}$

$v^5 = 0$ $\qquad u^5 = 1004e^{4x}$

$$y^5 = u^5v + nu^4v^1 + \frac{n(n-1)}{2!}u^3v^2 + \frac{n(n-1)(n-2)}{3!}u^2v^3 + \frac{n(n-1)(n-2)(n-3)}{4!}u^1v^4$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}uv^5$$

$$y^5 = [1024e^{4x}(x^3)] + [5(256e^{4x})3x^2] + [\tfrac{5\times4}{2}\times 64e^{4x}\times 6x] + [\tfrac{5\times4\times3}{3\times2}\times 16e^{4x}\times 6] + 0$$

$$y^5 = 1024e^{4x}x^3 + 1280e^{4x}(3x^2) + 640e^{4x}(6x) + 160e^{4x}(6)$$

$$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

i) $\frac{x^2d^2y}{dx^2} + \frac{x\,dy}{dx} + y = 0$

$$x^2y'' + xy + y = 0$$

ii) x^2y'' $\quad\quad v = x^2$

$u = y''$ $\quad\quad v' = 2x$

$u^n = y^n + 0$ $\quad\quad v'' = 2$

$v''' = 0$

$$y^n = u^nv + nu^{n-1}v' + n(n-1)u^{n-2}v'' + \frac{n(n-1)(n-1)}{3!}u^n = 3$$

$$y^n = y^{n+2}x^2 + ny^{n+1}(2x) + \frac{n(n-1)}{2!}y^n(2) + 0$$

$$y^n = y^{n+2}x^2 + 2xny^{n+1} + n(n-1)y^n$$

iii) xy'

$u = y'$ $\quad\quad v = x$

$u^n = y^{n+1}$ $\quad\quad v' = 1$

$v'' = 0$

$$y^n = u^nv + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v''$$

$$y^n = y^{n+1}x + ny^n(1) + 0$$

$$y^n = xy^{n+1} = ny^n$$

3) y

$u = y$ $\qquad v = 1$

$u^n = y^n$ $\qquad v^1 = 0$

$y^n = u^n v + n u^{n-1} v^1$

$= y^n(1) + n(y^{n-1})(0)$

$y^n = y^{n+2} x^2 + n y^{n+1} 2x + (n(n-1)) y^n + x y^{n+1} + n y^n + y^n$

$y^n = x^2 (y^{n+2}) + 2xn (y^{n+1}) + (n^2 - n) y^n + x(y^{n+1}) + n y^2 + y^n$

$y^n = (n^2 - n + n + 1) y^n + (2xn + x) y^{n+1} + x^2 (y^{n+2})$

$0 = x^2 (y^{(n+2)} + (2xn + x) y^{n+1} + (n^2+1) y^n$

$0 = x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n$

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$