$= x^2 y^{n+2} + 2xn(y^{n+1}) + n(n-1)(y^n)$

Let $w = xy'$

$v = x,\ v' = 1\ \ v'' = 0$

$u = y'\ \ \ u^n = y^{n+1}$

$w^n = y^{n+1}(x) + n(y^{n+1-x})(1) + 0$

$= xy^{n+1} + ny^n$

Let $w = y$

$v = 1\ \ v' = 0$

$u = y\ \ u^n = y^n$

$w^n = y^n$

$y^n = x^2 y^{(n+2)} + 2xn(y^{n+1}) + n(n-1)(y^n) + xy^{n+1} + ny^n + y$

$x^2 y(n+2) + (2n+1)xy^{(n+1)} + (n^2 - n + n + 1)y^n = 0$

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$ //

Let $w = -2y$

$v = -2 \quad v' = 0$

May $\quad u^n = y^n$

$w^n = u^n v + n u^{n-1} v'$

$= y^n \cdot -2 + 0$

$= -2y^n$

$\therefore y^n = y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^n$

$y^{n+2} - y^{n+1}(2x+1) + 2n(-y^n) - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^n(n-1)$

2i) $y = x^3 e^{4x}$ find y^5

$v = x^3,\ v' = 3x^2,\ v'' = 6x,\ v''' = 6,\ v^4 = 0$

$u = e^{4x},\ u' = 4e^{4x},\ u^2 = 16e^{4x},\ u^3 = 64e^{4x},\ u^4 = 256e^{4x},\ u^5 \ldots$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3$

$\frac{n(n-1)(n-2)(n-3)\, u^{n-4} v^4}{4!}$

$y^5 = u^5 v + 5u^4 v' + \frac{5(4) u^3 v^2}{2 \times 1} + \frac{5(4)(3) u^2 v^3}{3 \times 2 \times 1} + 0$

$y^5 = 1024 e^{4x}(x^3) + 15x^2(256 e^{4x}) + 60x(64e^{4x}) + 60(16e^{4x})$

$y^5 = x^3 1024 e^{4x} + x^2 3840 e^{4x} + x 3840 e^{4x} + 960 e^{4x}$

$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$

2ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$\Rightarrow x^2 y'' + xy' + y = 0$

Let $w = x^2 y''$

$v = x^2,\ v' = 2x \quad v'' = 2 \quad v''' = 0$

$u = y'' \quad u^n = y^{n+2}$

$w^n = u^n v + n u^{n-1} v' + \frac{n(n-1) u^{n-2} v''}{2!} + \frac{n(n-1)(n-2) u^{n-3} v'''}{3!}$

$= y^{n+2}(x^2) + n(y^{n+2-1})(2x) + \frac{n(n-1)}{2}(y^{n+2-2})(2)$

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15ICNG0E1002
Mechanical

1.) $y = e^{x^2+x}$, Show that $y'' = y'(2x+1) + 2y$

Solution

$y'' = y(2x+1) + 2y$

$y'' = \frac{d^2y}{dx^2}$ $y' = \frac{dy}{dx}$

$\therefore dy/dx = (2x+1)e^{x^2+x}$

using product rule: $v\,du/dx + u\,dv/dx$

$d^2y/dx^2 = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1)e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+3)e^{x^2+2x}$

Substituting RHS $ LHS above

$(4x^2+4x+3)e^{x^2+x} = (2x+1)e^{x^2+x}(2x+1) + 2(e^{x^2+x})$

$= (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$= (4x^2+4x+1+3)e^{x^2+x}$

$= (4x^2+4x+4)e^{x^2+x}$

$(4x^2+4x+3)e^{x^2+x} = (4x^2+4x+3)e^{x^2+x}$

$\therefore y'' = y(2x+1) + 2y$

ii.) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

le $W = y''$

$V = 1$ $V' = 0$

$U = y''$ $U^n = y^{n+2}$

$W^n = U^n V + nU^{n-1}V'$

$= y^{(n+2)} + 0$

Let $W = -y'(2x+1)$

$V = 2x+1$ $V' = 2$ $V'' = 0$

$U = -y'$ $U^n = -y^{n+1}$

$W^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V''$

$= -y^{n+1}(2x+1) + n(-y^{n+1})(2) + 0$

$= -y^{n+1}(2x+1) + 2n(-y^n)$