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15/ENG05/006
MECHATRONICS ENGR.

ENG 381 ASSIGNMENT III

1.) Question : $y = e^{x^2+x}$

Show that $y'' = y'(2x+1) + 2y$ and prove that $y(n+2) = (2x+1)y(n+1) + 2(n+1)y^n$

Solution

$y = e^{x^2+x}$ ——— (1.) ; $y' = (2x+1)e^{x^2+x}$ ——— (2.)

$\therefore y^n = C_1^n e^{ax}$

where $u\frac{dv}{dx} + v\frac{du}{dx} = \frac{dy}{dx}$

$u = 2x+1$; $du/dx = 2$

$v = e^{x^2+x}$; $dv/dx = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

From eqn(1) and (2.)

$y'' - y'(2x+1) + 2y$

Let $w_1 = y'$ $\quad w = y'(2x+1)$

$u = y^2$

$y^n = y(2+n)$

$u = y(1)$ $\quad v = 2x+1$

$u^2 = \cancel{y(} y^{(1+n)}$ $\quad v' = 2$

$u^{n+1} = y^n$

$W_2 = 2y$

$u = y$ $\quad v = 2$

$u^n = y^{(n)}$

$W_1^{(n)} = W_2^{(n)} + W_3^{(n)}$

$y^n = u^{(n)}v + nu^{(n-1)}v''$

$y^{(2+n)} = y^{(1+n)} \cdot (2x+1) + n(y(n)) \cdot 2 + y(n)2$

$y^{(2+n)} = (2x+1)y^{(1+n)} + 2(n+1)y(n)$

~~(n)~~ =

2) $y = x^3 e^{4x}$, determine $y^{(5)}$

Solution

$u = e^{4x}$

$u^n = 4^n e^{4x}$

$u^{n-1} = 4^{n-1} e^{4x}$

$u^{n-2} = 4^{n-2} e^{4x}$

$u^{n-3} = 4^{n-3} e^{4x}$

$y^{(n)} = u^n v + n u^{(n-1)} v' + \frac{n(n-1) u^{(n-2)}}{2!} v'' + \frac{n(n-1)(n-2) u^{(n-3)}}{3!} v'''$

$\cancel{y^{(5)} = 4^5 e^{4x} \cdot x^3 +}$

$y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x}) \cdot 3x^2 + \frac{5(4)(4^3 e^{4x}) \cdot 6x}{2!} + \cdots +$

$\frac{5(4)(3)(4^2 e^{4x}) \cdot 6}{3!} + \ldots$

$y^{(5)} = 1024x^3 + 3840x^2 e^{4x} + 3840x e^{4x} + 96 e^{4x}$

$y^{(5)} = 64 e^{4x} [16x^3 + 60x^2 + 60x + 15]$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

such that $x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1) y^n = 0$

Solution

Let $w = x^2 y$

$u = y''$, $u' = y'''$, $u'' = y^{iv}$ .... $u^{(n)} = y^{(n+2)}$

$v = x^2$

$v' = 2x$

$v'' = 2$

$v''' = 0$

$w^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + n(n-1)(n-2) u^{n-3} v'''$

$+ \ldots$

$w^n = y^{n+2} x^2 + n \cdot y^{n+1} \cdot 2x + \frac{n(n-1) y^n}{2} \cdot 2 + 0$

$W^n = y^{n+2} \cdot x^n + n \cdot 2xy^{n+1} + n(n-1)y^n$.

Let $w = xy'$

$u = y'$; $u' = y''$; $u^n = y^{(n+1)}$

$V = x$

$V' = 1$

$V'' = 0$

$$w(n) = u^n v^0 + \frac{n u^{n-1} v'}{1} + \frac{n(n-1) u^{n-2} v''}{2!} + \ldots$$

$$w^{(n)} = y^{n+1} + x + ny^n \cdot 1 + \frac{n(n-1)y^{n-1} \cdot 0}{2!}$$

$w(n) = y^{n+1} + x + ny^n$

where $\cancel{w} = y$

$W^2 = y^n$

$\therefore y^{n+2} x^2 + n \cdot y^{n+1} \cdot 2x + n(n-1)y^n + y^{n+1} + \cancel{x} + ny^n + y^n = 0$

$= y^{n+2} x^2 + [n \cdot 2xy^{n+1} + xy^{n+1}] + [n(n-1)y^n + ny^n + y^n] = 0$

$= x^2 y^{n+2} + (2x+1)xy^{n+1} + (n^2 - n + 1 + n)y^n = 0$

$= x^2 y^{n+2} + (2x+1)xy^{n+1} + (n^2+1)y^n = 0$

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