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ENG 381 Engineering Mathematics
Answers

Using the Leibnitz Theorem given that $y = x^3 e^{4x}$ determine $y^{(5)}$

Soln.

For Leibnitz Theorem

$$y = u^5 v + 5u^4 v' + 10u^3 v^2 + 10u^2 v^3 + 5u v^4 + v^5$$

Where

$u = e^{4x}$	$v = x^3$	1
$u' = 4e^{4x}$	$v' = 3x^2$	1 1
$u'' = 16e^{4x}$	$v'' = 6x$	1 2 1
$u''' = 64e^{4x}$	$v''' = 6$	1 3 3 1
$u^{(iv)} = 256e^{4x}$	$v^{(iv)} = 0$	1 4 6 4 1
$u^{(v)} = 1024e^{4x}$	$v^{(v)} = 0$	1 10 10 5 1

$$y^{(5)} = 1024e^{4x}(x^3) + 5(256e^{4x})(3x^2) + 10(64e^{4x})(6x) + 10(16e^{4x}) + 5(4e^{4x})(0) + (0)^5$$

$$y^5 = 1024x^3 \cdot e^{4x} + 3840x^2 \cdot e^{4x} + 3840x^2 + 960e^{4x}$$

$$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x^2 + 960)$$

If $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

Soln

let $w = x^2 y''$, $w_2 = xy'$, $w_3 = y$

for w_1

$u = y^{(2)}$	$v = x^2$
$u^n = y^{(n+2)}$	$v' = 2x$
	$v'' = 2$
	$v''' = 0$

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$u^{(n+1)} = y^{(n-1+2)} = y^{(n+1)}$
$u^{(n-1)} = y^{(n)}$

$w_1 = y^{(n+2)} - x^2 + ny^{(n+1)} 2x + \frac{n(n-1)}{2!} y^n = 0$

$w_2 = xy'$
$w_2 = y^{(n+1)} - x + ny^{(n)} \cdot 1 + 1$
$w_3 = y^n$

$w_1 + w_2 + w_3$

$x^2 y^{(n+2)} + ny^{(n+1)} \cdot 2x + \frac{n(n-1)}{2} \times 2xy^n + x y' + y^{(n+1)} + ny^{(n)} + y^n = 0$

$x^2 y^{(n+2)} + xy^{(n+1)} (2n+1) + y^{(n)} n(n-1) + (n+1) = 0$
$x^2 y^{(n+2)} + (2n+1) xy^{(n+1)} + (n^2 - n + 1) y^n = 0$
$x^2 y^{(n+2)} + (2n+1) xy^{(n+1)} + (n^2+1) y^{(n)} = 0$

1) If $y = e^{x^2+x}$, show that $y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^n$

Soln

$y = e^{x^2+x}$
Let $u = x^2 + x$
$\frac{du}{dx} = 2x+1$

$y = e^u$
$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

If $y' = e^{x^2+x} (2x+1)$

Thus

$y'' = e^{x^2+x}(2) + [e^{x^2+x} \cdot (2x+1)'] \, 2x+1$
$y'' = 2 \cdot e^{x^2+x} + e^{x^2+x} \cdot (2x+1) \cdot (2x+1)$

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but $y = e^{x^2+x}$

and $y' = e^{x^2+x}(2x+1)$

$y'' = 2(y) + y'(2x+1)$ QED

Applying leibnitz theorem to the above equation finding ths nth derivative

$y^{(n+2)} = 2(n+1)y^{n} + y^{(n+1)}(2x+1)$