ELENDU DIVINE
15/ENGO6/xx
MECHANICAL ENGINEERING
ENG 381

1 $y = e^{x^2} + x$

$u = x^2 + x$

$du/dx = 2x + 1$

$y = e^u$

$dy/du = e^u$

$dy/dx = dy/du \times du/dx$

$= e^u \times 2x + 1$

$2x + 1e^u \quad u = x^2 + x$

$dy/dx = 2x + 1e^{x^2+x}$

$d^2y/dx^2 = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$d^2y/dx^2 = 2e^{x^2+x} + 4x^2 + 4x + 1 + e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} \quad y' = \frac{dy}{dx} \quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y' = (2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + 1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$\underset{Q_1}{y''} = \underset{Q_2}{y'(2x+1)} + \underset{Q_3}{2y}$

Q_1 ; $u = y'' \qquad v = 1$

$u^n = y^{n+2} \qquad v = 0$

$= y^{n+2} \cdot 1 + 0$

Q_2 : $u = y', u^n = y^n \quad v = 2x + 1$

$u^n = y^{n+1} \qquad v' = 2 \quad v = 0$

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$= y^{n+1}(2x+1) + n(y^n)2 + 0$

$y^{n+1}(2x+1) + 2n(y^n)$

ω_3: $u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v = 0$

$= 2(y^n \cdot 1 + 0)$

$= 2y^n$

$\omega_1 = \omega_2 + \omega_3$

$y^{n+1} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2a Using the leibnitz theorem gives that

$y = x^3 e^{4x}$ determine $y^{(5)}$

Soln

$u = e^{4x}$ $\quad v = x^3$

$y^{(5)} = u^{(5)}v + 5u^4v^1 + 10u^3v^2 + 10u^2v^3 + 5u^1v^4 + uv^5$

$= 4^5 e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} \cdot 6x) + 10(4^2 e^{4x} \cdot 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 640 e^{4x} 6x + 160 e^{4x} \cdot 6$

$= 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 960 e^{4x}$

b $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

$\underbrace{x^2 y''}_{\omega_1} + \underbrace{xy'}_{\omega_2} + \underbrace{y}_{\omega_3} = 0$

$\omega_1 + \omega_2 + \omega_3 = 0$

For ω_1

$u = y''$ $\quad v = x^2$

$u^n = y^{n+2}$ $\quad v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad v'' = 2$

$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \dfrac{n(n-1)}{2!} y^n \cdot 2 = 0$

$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$

For ω_2

$u = y'$ $\quad v = x$

$u^n = y^{n+1}$ $\quad v = 1$

$u^{n-1} = y^n$ $\quad v' = 0$

$= y^{n+1} \cdot x + ny^n + 0$

For ω_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$= y^n \cdot 1$

$\omega_1 + \omega_2 + \omega_3 = 0$

$= x^2 y^{n+2} + 2nxy^{n+1} + (n^2 - n)y^n + xy^{n+1} + ny^n + y^n$

$= xy^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2y^n - \cancel{ny^n} + \cancel{ny^n} + y^n$

$= x^2 y^{n+2} + 2n+1\,(xy^{n+1}) + (n^2+1)y^n$