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If $y = e^{x^2+x}$

show that $y'' = y'(2x+1) + 2y$

Soln

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

recall

$y = e^{x^2+x}$ and $y' = (2x+1)e^{x^2+x}$

then

$y'' = (2x+1) \cdot y' + 2 \cdot y$

$\therefore y'' = y'(2x+1) + 2y$

Hence prove that

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

using leibnitz theorem

$y'' = y'(2x+1) + 2y$

$y'' + y'(2x+1) + 2y = 0$

v_1 v_2 v_3

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$u_1 = y'' = d^2y/dx^2$

then

$u = y''$ $\qquad v = 1$

$u^n = y^{(n+2)}$ $\quad v' = 0$

$u_2 = y'(2x+1)$

$u = y'$ $\qquad v = 2x+1$

$u^n = y^{(n+1)}$ $\quad v' = 2$

$v'' = 0$

$u_3 = 2y$

$u = y$ $\qquad v = 2$

$u^n = y^n$ $\quad v' = 0$

combining all three

Therefore

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2ny^{(n)} + 2y^{(n)}$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)_2$

② Using arbitrary theorem

$y = x^3 e^{4x}$

determine y^5

Solution

take $u = e^{4x}$ and $v = x^3$

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$$y^n = u^n v + n u^{(n-1)} v^n + \frac{n(n-1)}{2!} u^{(n-2)} v^2 .$$

$$+ \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \ldots$$

$u^n = 4^n e^{nx}$ $\qquad v = x^3$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\qquad v' = 3x^2$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\qquad v'' = 6x$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\qquad v''' = 6$

$$y^{(n)} = 4^n e^{4x} . x^3 + \cancel{t} n 4^{(n-1)} e^{4x} . 3x^2 + \frac{n(n-1)}{2!} 4^{(n-2)} e^{4x} . 6x \cancel{+ n(n-1)(n-2)}$$

$$+ \frac{n(n-1)(n-2)}{3!} 4^{(n-3)} e^{4x} . 6$$

$$y^{(n)} = x^3 4^n e^{4x} + 3x^2 n 4^{(n-1)} e^{4x} 3x n^{(n-1)} 4^{(n-2)} + n(n-1)(n-2) 4^{(n-3)} e^{4x}$$

$y^{(5)}$

$$= x^3 (4)^5 e^{4x} + 3x^2 (5) 4^{(5-1)} e^{4x} + 3x(5)(5-1) 4^{(5-2)} + 5(5-1)(5-2) 4^{(5-3)} e^{4x}$$

$$= 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x + 960 e^{4x}$$

$$= e^{4x} [1024 x^3 + 3840 x^2 + 3840 x] + 960$$