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15/ENG01/008
CHEMICAL ENGINEERING
ENG381

1. If $y = e^{x^2+x}$

$u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$ $\quad dy/du = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u \times 2x + 1$

$2x+1e^u$

when $u = x^2 + x$

$\frac{dy}{dx} = 2x + 1e^{x^2+x}$

$\frac{d^2y}{dx^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$= 2e^{x^2+x} + 4x^2 + 4x + e^{x^2+x}$

$d^2y/dx^2 = 2e^{x^2+x} + 4x^2 + 4x + e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2}$ $\quad y' = dy/dx$ $\quad y = e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1\, e^{x^2+x}$

$y'(2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + e^{x^2+1}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1 + e^{x^2+x}$

$y'' = \cancel{2x}\ 2e^{x^2+x} + 4x^2 + 4x + e^{x^2+x}$

$W_1 = y''$

$W_2 = y'(2x+1)$

$W_3 = 2y$

For W_1

$u = y''$ $\quad v = 1$

$u^n = y^{n+2}$ $\quad v = 0$

$= y^{n+2} \cdot 1 + 0$

For W_2

$u = y'$ $\quad v = 2x+1$

$u^n = y^{n+1}$ $\quad v' = 2$

$u^{n-1} = y^n$ $\quad v = 0$

$= y^{n+1}(2x+1) + n(y^n) \cdot 2 + 0$

$= y^{n+1}(2x+1) + 2n(y^n)$

For W_3

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v' = 0$

$2[(y^n \cdot 1) + 0]$

$2y^n$

$W_1 = W_2 + W_3$

$y^{n+2} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2. i Using the Leibnitz theorem given that

$y = x^3 e^{4x}$

Determine $y^{(5)}$

$V = x^3$ $\quad$ $u = e^{4x}$

$V' = 3x^2$ $\quad$ $u' = 4e^{4x}$

$V'' = 6x$ $\quad$ $u'' = 16e^{4x}$

$V''' = 6$ $\quad$ $u''' = 64e^{4x}$

$V^{(4)} = 0$ $\quad$ $u^{(4)} = 256e^{4x}$

$V^{(5)} = 0$ $\quad$ $u^{(5)} = 1024e^{4x}$

$$y^{(5)} = u^5 V + n \cdot u^4 V' + \frac{n(n-1)}{2!} u^3 V^2 + \frac{n(n-1)(n-2)}{3!} u^2 V^3 + \frac{n(n-1)(n-2)(n-3)}{4!} u' V^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} \cdot u V^5$$

$$y^{(5)} = [1024e^{4x}(x^3)] + [5(256e^{4x})3x^2] + \left[\frac{5\times4}{2} \times 64e^{4x} \times 6x\right] + \left[\frac{5\times4\times3}{3\times2} \times 16e^{4x} \times 6\right] + [0][0]$$

$$y^{(5)} = 1024e^{4x}x^3 + 1280e^{4x}(3x^2) + 640e^{4x}(6x) + 160e^{4x}(6).$$

$$y^{(5)} = 1024e^{4x} \cdot x^3 + \cancel{1280e^{4x}(3x^2)}\ 3840e^{4x} \cdot x^2 + 3840e^{4x} \cdot x + 960e^{4x}$$

$$y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{(n-2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$x^2 y'' + xy' + y = 0$

$w_1 = x^2 y''$ $\quad$ $w_2 = xy'$ $\quad$ $w_3 = y$

For W_1

$\cancel{u=y^n}$

$u = y''$ $\quad$ $v = x^2$

$u^n = y^{n+2}$ $\quad$ $v' = 2x$

$u^{n-1} = y^{n+1}$ $\quad$ $v'' = 2$

$u^{n-2} = y^n$ $\quad$ $v''' = 0$

$$= y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)y^n \cdot 2}{2!} + 0$$

$$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$$

For W_2

$u = y'$ $\quad$ $v = x$

$u^n = y^{n+1}$ $\quad$ $v' = 1$

$u^{n-1} = y^n$ $\quad$ $v'' = 0$

$$= y^{n+1} \cdot x + ny^n + 0$$

For W_3

$u = y$ $\quad$ $v = 1$

$u^n = y^n$ $\quad$ $v' = 0$

$$= y^n \cdot 1$$

$W_1 + W_2 + W_3 = 0$

$$x^2y^{n+2} + 2nxy^{n+1} + (n^2-n)y^n + xy^{n+1} + ny^n + y^n$$

$$x^2y^{n+2} + 2nxy^{n+1} + xy^{n+1} + n^2y^n - ny^n + ny^n + y^n$$

$$x^2y^{n+2} + 2n+1(xy^{n+1}) + (n^2+1)y^n$$