UNDIE FIDELIS BEWHAPNYE
CIVIL ENGINEERING
ENG 381
15/ENG03/033

1) $y = e^{x^2+x}$

let $v = x^2 + x$

$\frac{dv}{dx} = 2x + 1$

$y = e^v$

$\frac{dy}{dv} = e^v$

$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx}$

$= e^v \times 2x + 1$ where $v = x^2 + x$

$\frac{dy}{dx} = e^{x^2+x} \cdot (2x+1) = y'$

If $y' = e^{x^2+x} \cdot (2x+1) = y'$

then $y'' = 2e^{x^2} \cdot e^{x^2+x}(2) + [e^{x^2+x} \cdot (2x+1)](2x+1)$

$y'' = 2e^{x^2+x} + e^{x^2+x} \cdot (2x+1) \cdot (2x+1)$

but $y = e^{x^2+x}$

and $y' = e^{x^2+x}(2x+1)$

$y'' = 2(y) + y'(2x+1)$

Applying Lebnitz theorem to the above equation. Find the nth derivative

$y^{(n+2)} = 2(n+1)y^n + y^{(n+1)}(2x+1)$

2) Using Lebnitz theorem given that

$y = x^3 e^{4x}$ determine $y(5)$

Soln

Recall that Lebnitz theorem states that

$y^5 = u^5 v + 5u^4 v + 10u^3 v^2 + 10u^2 v^3 + 5u' v^4 + v^5$

where $u = e^{4x}$ $\quad v = x^3$

$u^1 = 4e^{4x}$ $\quad v^1 = 3x^2$

$u^2 = 16e^{4x}$ $\quad v^2 = 6x$

$u^3 = 64e^{4x}$ $\quad v^3 = 6$

$u^4 = 256e^{4x}$ $\quad v^4 = 0$

$u^5 = 1024e^{4x}$

Therefore

$$y^5 = 1024e^{4x}(x^3) + 5(256e^{4x})(3x^2) + 10(64e^{4x})(6x) + 10(16e^{4x})6 + 5(4e^{4x})(0)$$

$$= 1024x^3 \cdot e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$$

$$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

ii) If $x^2\frac{dy}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Soln

The equation can be written as $x^2y'' + xy' + y = 0$

let $w_1 = x^2y''$, $w_2 = xy'$, and $w_3 = y$

Solving for $W_1 = x^2y''$

let $u = y^2$, $U^n = y^{n+2}$

let $v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$$W = y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{2!}y^n \times 2 + 0$$

$w_2 = xy'$

let $u = y'$, $u^n = y^n$

let $v = x$, $v' = 1$ and $v'' = 0$

$$w_2 = y^{(n+1)} \cdot x + ny^n \times 1 + 0$$

$$= xy^{n+1} + ny_n$$

$w_3 = y^{(n)}$

Combining

$w = w_1 + w_2 + w_3$

$$w = x^2 y^{(n+2)} + n y^{(n+1)} \cdot 2x + \frac{n(n-1)}{2} y^{n<2} \cdot x y' + y^{(n+1)} x + n y^n + y^n = 0$$

$$= x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + y^n \left[(n(n-1) + n + 1 \right] = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{n+1} + (n^2+1) y^n = 0$$