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15/ENG06/042
MECHANICAL ENGINEERING
ENG 381

ASSIGNMENT

(1) $Y = e^{x^2+x}$

Soln

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$y'' = 2y + (2x+1)y'$

hence

$y'' = y'(2x+1) + 2y$

$y^{(n+2)} = y^{(n+1)} \cdot (2x+1) + (n+1)y^n \cdot 2$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

(2) $y = x^3 e^{4x}$

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)v^3}{3!} + \dots + t$

Let $x^3 = v$ and $u = e^{4x}$

$y^5 = (4)^5 e^{4x} \cdot x^3 + 5(4)^4 e^{4x} \cdot 3x^2 + 10(4)^3 e^{4x} \cdot 6x + 10(4)^2 e^{4x} \cdot 6 + 0$

$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x} + 0$

$y^5 = e^{4x}[1024x^3 + 3840x^2 + 3840x + 960]$

3) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$x^2 y_{(2)} + xy' + y = 0$

Using leibnitz theorem.

$y^n = y_{(n+2)} \cdot x^2 + n \cdot 2x\, y_{(n+1)} + \frac{2n(n-1)y^n}{2!} + y_{(n+1)} \cdot x + n y^n \cdot 1 + y^n$

$y^n = x^2 y_{(n+2)} + 2x\, y_{(n+1)} n + n(n-1) y^n + x y_{(n+1)} + n y_{(n)} + y_{(n)}$

$y^n = y_{(n+2)}(x^2) + y_{(n+1)}(2xn + x) + y_{(n)}(n(n-1) + n + 1)$

$y^n = x^2 y^{n+2} + (2n+1)\, x y^{n+1} + (n^2-1) y^n$

$x^2 y_{(n+2)} + (2n+1)\, x y_{(n+1)} + (n^2-1) y^n = 0$