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Matric No: 16/ENG06/082 300 level

i) If $y = e^{x^2+x}$

Show that

$y'' = y'(2x+1) + 2y$

Soln

$y = e^{x^2+x}$

$\frac{dy}{dx} = y'$

Let $u = x^2+x$

$\frac{du}{dx} = 2x+1$

$\therefore y = e^u$; $\frac{dy}{dx} = y' = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = y' = e^u \times 2x+1$

$y' = [2x+1]e^u$

$y' = [2x+1]e^{x^2+x}$

$\frac{d^2y}{dx^2} = y''$

$y' = [2x+1]e^{x^2+x}$

using product rule

let $u = 2x+1$ $v = e^{x^2+x}$

$\frac{du}{dx} = 2$

using chain rule

$\frac{dv}{dx} = 2x+1e^{x^2+x}$

$\therefore \frac{d^2y}{dx^2} = y'' = v\,{}^{du}/_{dx} + u\,{}^{dv}/_{dx}$

$$= 2\left[e^{x^2+x}\right] + 2x+1\left[2x+1e^{x^2+x}\right]$$

since, $y = e^{x^2+x}$

$y' = [2x+1]e^{x^2+x}$

$y'' = 2[y] + (2x+1)y'$

$y''' = 2y + (2x+1)y'$

$y^{iv} = y'(2x+1) + 2y$

b) Hence prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

Solution

$$y'' = y'(2x+1) + 2y$$

$$y'' - y'(2x+1) - 2y = 0$$

using leibnitz theorem

i) y''

$u = y''$; $v = 1$

$u = y^{n+2}$ $v' = 0$

$= u^n v + n u^{n-1} v'$

$= y^{n+2}(1) + n(y^{n+1})(0)$

$= y^{n+2}$

ii) $y'(2x+1)$

$u = y'$ $v = -(2x+1)$

$u^n = y^{n+1}$ $v' = -2$

$v'' = 0$

$= u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v''$

$= -y^{n+1}(2x+1) + n(y^n)(-2)$

$= -(2x+1)y^{n+1} - 2ny^n$

iii) $-2y$

$u = y$ $v = -2$

$u^n = y^n$ $v' = 0$

$= u^n v + n u^{n-1} v'$

$= -2y^n$

$\therefore y^{n+2} - (2x+1)y^{n+1} - 2ny'' - 2y^n = 0$

$\therefore y^{n+2} = (2x+1)y^{n+1} + 2ny'' + 2y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + 2y^n(n+1)$

$y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

b)

2.) $y = x^3 e^{4x}$

y^5

$v = x^3$ $\quad$ $u = e^{4x}$

$v' = 3x^2$ $\quad$ $u' = 4e^{4x}$

$v^2 = 6x$ $\quad$ $u^2 = 16e^{4x}$

$v^3 = 6$ $\quad$ $u^3 = 64e^{4x}$

$v^4 = 0$ $\quad$ $u^4 = 256e^{4x}$

$v^5 = 0$ $\quad$ $u^5 = 102 e^{4x}$

$$y^5 = u^5v + nu^4v' + \frac{n(n-1)u^3v^2}{2!} + \frac{n(n-1)(n-2)u^2v^3}{3!} + \frac{n(n-1)(n-2)(n-3)u'v^4}{4!}$$

$$+ n(n-1)(n-2)(n-3)(n-4)uv^5/5!$$

$$y^5 = 1024e^{4x}x^3 + 1280e^{4x}(3x^2) + 640e^{4x}(6x) + 160e^{4x}(0)$$

$$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy + y = 0$

a) x^2y''

$u = y''$ $\quad$ $v = x^2$

$u^n = y^{n+2}$ $\quad$ $v' = 2x$

$v'' = 2$

$v''' = 0$

$$y^n = u^nv + nu^{n-1}v + \frac{n(n-1)u^{n-2}v''}{2!} + \frac{n(n-1)(n-2)u^{n-3}v'}{3!}$$

$$y^n = y^{n+2}x^2 + 2xny^{n+1} + n(n-1)y^n$$

b) xy'

$u = y'$ $\quad$ $v = x$

$u^n = y^{n+1}$ $\quad$ $v' = 1$

$v'' = 0$

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$$y^n = u^n + nu^{n-1}v' + \frac{n(n-1)}{2!} u^{n-2}v''$$

$$y^n = y^{n-1}x + ny^n(1) + 6$$

$$y^n = xy^{n+1} + ny^n$$

c) y

$u = y \qquad v = 1$

$u^n = y^n \quad v' = 0$

$y^n = u^n v + nu^{n-1}v'$

$= y^n(1) + n(y^{n-1}) \cancel{x} (0)$

$$y^n = y^{n+2}x^3 ny^{n+1} 2x + (n(n-1)y^n) + xy^{n+1} + ny^n + y^n$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$$

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